Carbon Emissions Resulting from Different Power Flow Models for Dispatch

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***Abstract*—Optimal power flow (OPF) seeks to minimize the cost of electric power generation subject to physical constraints. The objective function in the OPF problem is typically defined in terms of dollars, and not in terms of environmental objectives such as plant emissions. However, the mix of generators that result in the lowest system cost does not always fully correlate with the mix of generators that result in the lowest system emissions. This can be further exacerbated under a DC OPF framework, which utilizes slack bus generators (often fast-ramping gas plants) to ensure AC feasibility. This paper analyzes the difference in emissions under different power flow models to quantify how cost-based objectives in OPF have impacts on the resulting system emissions. The IEEE 118-bus system is considered as our initial test case, using demand data from CAISO. Our results indicate that the choice of slack bus(es) can heavily influence the difference in emissions. We also propose an alternative DC OPF framework which reduces utilization of the slack bus (thus reducing emissions) and avoids possible constraint violations.**

***Index Terms*—Optimal power flow, carbon emissions, constraint violations**

I. Introduction

The canonical optimal power flow problem (OPF) typically aims to minimize the cost of (active power) generation subject to system constraints. The complete AC optimal power flow formulation is challenging to compute on large power grids (which may have tens of thousands of buses). Therefore, approximate or relaxations of the formulation are typically used in practice (e.g. [1]). However, the choice to move to a non-exact power flow methods has well-documented consequences on system cost – which vary between networks [2], [3].

Ambitious goals have been set for decarbonization of the power sector – for example, the U.S. government has set a target of a zero carbon power grid by 2035 [4]. Achieving this will require policy changes to incentivize a shift away from carbon-intensive generation sources [5]. Currently, the cost considered in the OPF formulation does not directly account for carbon emissions produced from each generator. The carbon emissions per unit of electricity generated varies significantly between generator types. It is well-known that historically, the generators that are dispatched in OPF do not always correspond to the mixture of generators that result in the lowest level of system emissions [6].

This material is based upon work supported by the National Science Foundation under Grant No. 2041835.

Previous work has proposed new dispatch methods which take into account carbon emissions [7]–[9] or new pricing mechanisms such that prices are forced to better reflect emissions [10]. However, here we seek to quantify the difference in emissions that results between exact and non-exact methods for dispatch. This is distinct from these previous efforts, because we do not seek to directly influence carbon emissions, rather to better understand the consequences of using approximations for the full OPF problem.

Thus, in this paper, we analyze how the underlying power flow model within the OPF impacts resulting emissions from generation. In addition to comparing the AC OPF with a DC OPF (with AC power flow to ensure feasibility), we also develop a third alternative. This method still utilizes the typical DC OPF formulation, but includes a second stage ensuring the solutions is physically realizable with respect to the AC power flow constraints. As a secondary insight, we further consider constraint violations resulting from the standard DC OPF with AC power flow. These violations, in practice, typically result in an iterative procedure where the DC OPF is run multiple times with additional constraints.

II. Background

In contrast to other commodities such as water or oil, electric power cannot be easily stored and must be generated on demand. The imminence of electrical usage creates the need for an Independent System Operator (ISO) which is an organization that can monitor shifts in demand on the grid and respond in a safe, reliable, cheap and feasible manner. ISOs can accomplish this by iteratively solving Optimal Power Flow (OPF) problems. However, the non-linear, non-convex nature of the complete AC OPF presents challenges in practice – the most salient of these including convergence issues and calculation time.

To address these challenges, many ISOs use DC OPF approximation of the AC OPF problem and then subsequently solve an AC power flow (PF) to account for losses and ensure voltage constraints are met. While this method is beneficial in its relatively fast computation time and straightforward derivation of prices, it may produce sub-optimal solutions – potentially costing billions of dollars annually [11]. Additionally, the DC OPF results in dispatch solutions that do not satisfy system constraints [12], necessitating the need for subsequent AC PFs until AC feasibility is achieved. More robust solution techniques to the AC OPF problem have been developed over the past years, reviewed in [13], [14], that unlock the potential for ISOs to preferentially utilize AC OPF solutions rather than the DC OPF approximation. Here we study the consequences of the choice of OPF technique on the system carbon emissions.

III. Methods

In this section, we first introduce the methods by which the parameters of our test network are scaled to achieve realistic results. Then three generator dispatching methods are introduced, which each aim to minimize cost. Finally, the method for quantifying the emissions produced from each dispatch solution is explained.

A. Network scaling

We consider a test network with n_b buses, n_g generators, and n_l loads. In order to simulate a realistic loading scenario, fifteen minute demand data was taken from the California ISO (CAISO). We scale the network loads linearly, preserving the test networks' spatial load distribution. The factor, λ, by which the data was scaled down by is defined by:

$$\lambda = c * \frac{\sum_g^{n_g} P_g^{max}}{\rho_d^{peak}}, \tag{1}$$

where P_g^{max} is the max power in megawatts that generator g can produce and ρ_d^{peak} is the peak demand in megawatts in a 24 hour period of the CAISO data. The constant $0 < c < 1$ is introduced to simulate some level of oversupply (i.e. to ensure that peak demand is less than peak supply). Without this scaling factor, the optimization problem would not be able to converge near the peak demand because the system would not have the capability of providing the extra power for line losses. We assume that the power factor of the loads does not change, meaning that the reactive loads are scaled proportionally to the real loads. Thus, the re-scaled real and reactive loads l are given by:

$$P_{d\_lt} = \frac{P'_{dl}}{\sum_l^{n_l} P'_{dl}} * \lambda \rho_{d\_t} \tag{2}$$

$$Q_{d\_lt} = \frac{Q'_{dl}}{P'_{dl}} * P_{d\_lt} \tag{3}$$

Values P'_{dl} and Q'_{dl} are, respectively, the original active and reactive power demand (in megawatts) at load l from the standardized system. Similarly, P_{d_lt} and Q_{d_lt} are, respectively, the active and reactive power demanded at load l at time t. The variable ρ_{d_t} corresponds to the CAISO 15 minute demand data in megawatts at time t. Equation (2) ensures that the demand at a single load remains at a constant proportion to the total demand on the network. Likewise, equation (3) ensures that the ratio of active to reactive power remains constant.

B. Dispatching Methods

Once the network loads have been resized, we need to dispatch generation which meets the demand at lowest cost, while respecting system constraints. Here, three separate methods are considered.

1) AC Optimal Power Flow: The first method we consider is using AC optimal power flow (AC OPF), which is assumed to be the "ground-truth" model for how the power system behaves. This method generally considers a quadratic cost objective for generation, such that we aim to minimize:

$$f(P_g) = \sum_g c_{g_0} + c_{g_1} P_g + c_{g_2} P_g^2, \tag{4}$$

where c_{g_1}, c_{g_2} are the linear and quadratic cost terms of generator g respectively, and c_{g_0} are fixed costs. The constraints of the problem are the AC power flow constraints [14] which include constraints on bus voltage magnitude and thermal limits of lines and transformers. The thermal limits include both quadratic and multiplicative terms, thus resulting in a non-convex problem.

2) DC Optimal Power Flow: The second method we considered was DC optimal power flow (DC OPF) a commonly used first order approximation of the AC OPF problem which has been shown to provide a reasonable representation of the power system under normal conditions [15]. Here we consider the same objective function (4) as above. One of the major assumptions of the DC OPF problem is the use of a linear approximation for the thermal limits while neglecting the voltage limits (as variables for voltage magnitude are not included). This means that the solution of the DC OPF necessarily does not satisfy the AC power flow constraints [12]. However, due to its convexity and significant computational benefits over AC OPF, this method is commonly used in practice. In order to solve the AC infeasibility of our DC OPF solution we run a power flow problem, where the generator outputs are fixed and a Newton-Raphson method is used to find a solution which satisfies the AC power flow constraints. In order to compensate for the difference in needed generation, a generator on the slack bus of the network (or slack buses) are used.

3) DC Nearest Feasible Point: In this alternative method, we consider a dispatch which first uses DC OPF, but then finds the nearest AC feasible point. This method combines the dispatch decisions from the DC OPF problem, which are often used in ISO market clearings, with the physical feasibility of the AC OPF. Additionally, it allows us to comment on how the performance of the DC OPF dispatch is effected by the lack of feasibility. Here, we run a second optimization which seeks to minimize distance from the DC OPF dispatch, hence minimizing:

$$f(P_g) = \Sigma_g^{n_g} (P_g - P'_g)^2 \tag{5}$$

where P'_g are the DC OPF generator outputs. Expanding this equation and disregarding the constant terms (which have no impact on the optimal solution) yields:

$$f(P_g) = \Sigma_g^{n_g} P_g^2 - 2P'_g P_g \,. \tag{6}$$

This objective is minimized subject to the AC OPF constraints, which can be performed in PandaPower by adjusting the generator constant and quadratic objectives.

C. Emissions Calculation

The generator output solutions are then used to determine the tons of CO2 equivalent (tCO2e) per fifteen minutes produced by the system at time t. These emissions are represented by ϵ_t below:

$$\epsilon_t = \sum_{g}^{n_g} \varepsilon_g P_{gt} * 0.25\,, \qquad (7)$$

where P_{gt} is the active power (in MW) generated by generator g at time t, ε_g is the emissions factor (in tons of CO2e per MWh) associated with generator g, and 0.25 converts MW into MWh. The emissions factor is determined by the type of generator (these are defined in the test cases). Here we use the standard emissions factors provided for each generator type from the National Renewable Energy Laboratory (NREL) [16].

IV. Results

Our methods are tested using a case study of the IEEE 118 test system and generator types from [17]. Solutions were computed using the Pandapower [18] toolbox in Python, which uses a Newton-Raphson-based iterative method. We use the CAISO demand for the 24-hour period on June 30th, 2022, and consider the three dispatching methods described in the previous section.

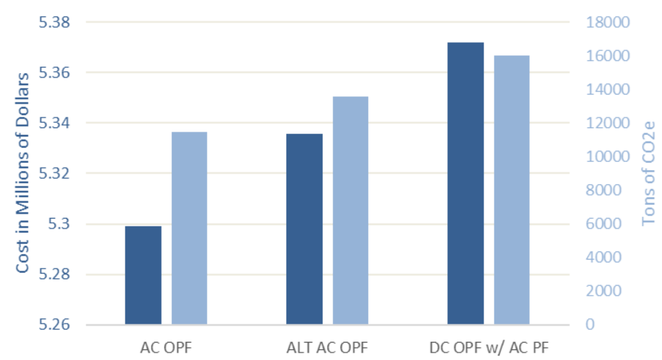

Fig. 1: Generation costs and emissions over a single day simulation for each of the three dispatch methods considered.

Figure 1 shows the total generator cost (navy blue) and CO2 emissions (light blue) that resulted from each of the three dispatch methods. As was to be expected, the AC OPF has the lowest cost solution – this was the only formulation which did not have a secondary adjustment stage. It also has the lowest emissions, which might be because many of the cheapest generator types are also those with the lowest level of carbon emissions.

The other two methods both initially use DC OPF to determine the generator outputs. The difference between them is the way in which they handle the infeasiblity of the DC constraints. The DC OPF with AC power flow method fixes generator outputs and utilizes the slack bus generator to solve any power imbalances. Whereas, the modified AC OPF alters all of the generator outputs to find the closest feasible point. Figure 1 demonstrates that the latter is both the cheaper and lower carbon alternative. This is because for the 118-bus test case the slack generator is coal, which is both high cost and high carbon. Slack generators are chosen that can ramp up quickly, meaning that a dispatchable, non-renewable slack generator is likely common. By distributing the required slack across all generators on the network, we do not require such steep ramp rates. This suggests that this second method should, more generally, result in lower emissions (rather than just in the 118 test case), since slower ramping generators can be used to correct for AC feasibility.

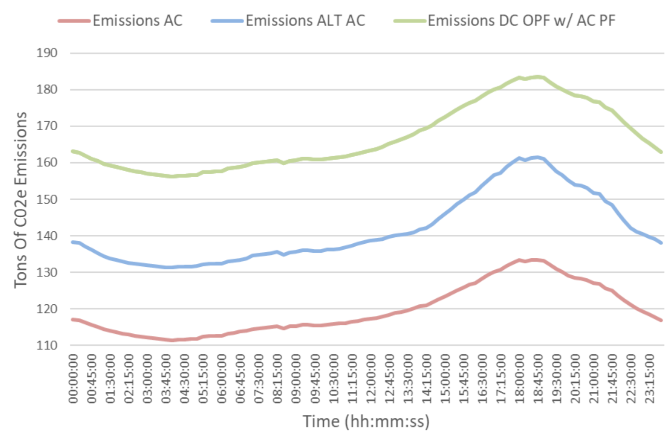

Fig. 2: Profiles of emissions at 15 min resolution throughout the 24h for each method.

Figure 2 shows how the emission produced by each method are distributed throughout the 24 hour period. We can see that the DC OPF methods are consistently higher emission than using AC OPF. This difference is most significant at peak demand time, which might be because the network is highly constrained at this time.

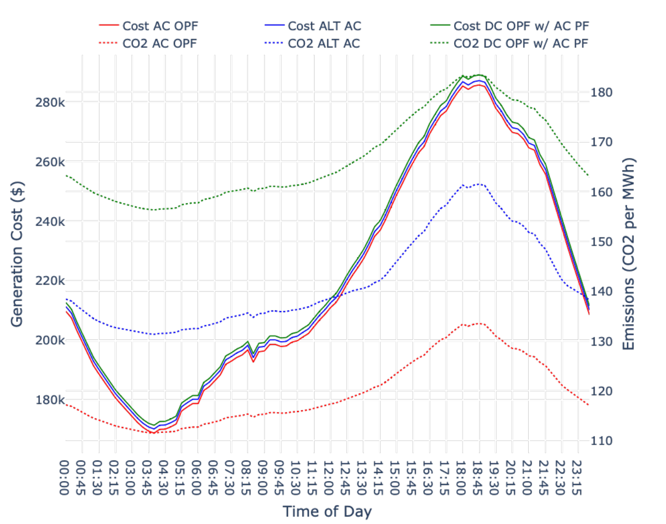

Fig. 3: Generation costs (left) and carbon emissions (right) per 15 minute period over a single day simulation for each of the three dispatch methods considered.

Additionally, in Fig. 3, CO2e emissions per MWh are provided. This reiterates the findings from Fig. 2, and provides further context that, even when a larger generation capacity is needed, dispatch methods using a power flow correction result in a higher emissions per MWh of generation. The cost per MW is also provided in Fig. 3, while the cost does not show significant differences per hour, the daily difference between AC OPF and DC OPF with AC PF results in a $\$290,992.66$ difference – and is expected to be higher for a higher demand day and larger network. Savings would additionally increase in real power grid operations due to the availability of running an AC OPF in real-time may reduce reserve margins that are high in emissions [19]. The cost gap is also expected to widen with a network incorporating renewable resources such as wind and solar, since the AC OPF would select the lowest cost generators. Using distributed slack buses may also have a significant impact on pricing and emissions.

Another aspect which is not taken into account in the cost and emissions analysis, is that the DC OPF with AC power flow correction does not ensure that the voltage constraints are met. While only varying one generator on the network and needing to meet a defined mismatch in supply and demand, there is not additional flexibility to protect voltage constraints. In Fig. 4, test-case 118 is shown at its peak demand for June, 30th (7972.96 MW) with the DC OPF and AC power flow correction shown on the left, and the AC OPF on the right. In the AC OPF case all bus voltages are within the required bounds, while in the DC OPF with AC correction there exist multiple buses with under-voltages. The DC OPF case also has a significantly larger voltage drop across branches (a much greater difference in color can be seen between two connected nodes). Given that most nodes are load buses, this demonstrates that the losses in the network would be much higher than in the AC OPF case. Therefore, using the DC OPF method may also result in unwanted power quality issues.

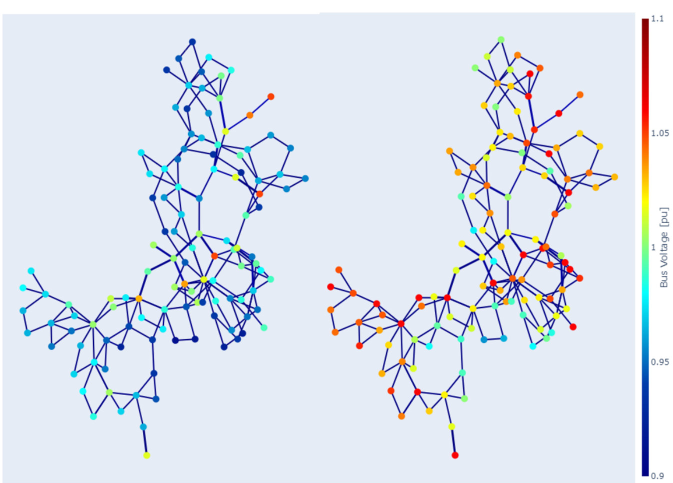

Fig. 4: A visual representation of voltage magnitude in the 118-Bus network at peak demand. Left is shown the DC OPF with AC power flow correction and right is shown the AC OPF.

TABLE I: Performance of the considered methods in terms of cost, emissions, voltage violations, and computation time.

Metric	AC OPF	AC ALT	DC OPF w AC
Emissions (Tons of CO2e)	11,490.61	13,603.82	16,005.74
Cost	\$21,196,158.05	\$21,342,705.07	\$21,487,150.71
Voltage Violations (#)	0	0	181
Computation Time (s)	2.25	2.24	0.25

Further, Fig. 5 shows the magnitude of voltage violations over the 24-hour period, as the height of the bars depicts the largest voltage violation on the network at each time step, the color of the bar depicts the number of voltage violations on the system. Again, this demonstrates that DC OPF with AC power flow correction has greater severity and quantity of voltage violations during peak demand hours when the network is strained. All of the voltage violations violate the minimum voltage constraint as opposed to the maximum voltage constraint. A summary of the number of violations, cost, and emissions are provided in Table I. It can be seen that AC OPF significantly outperforms DC OPF in all metrics. However, including the alternative objective rather than a standard power flow correction eliminates constraint violations, and achieves approximately half the possible reduction in both cost and emissions. Other violations investigated were line overloads, but no line overloading was observed for any of the three dispatch methods. We see that the DC OPF method results in the lowest computational cost, and we expect the cost savings to scale non-linearly with network size. We expect that at this network size, the benefits of the faster AC OPF are marginalized by needing to run two optimization schemes. However, for larger networks we would expect a more significant savings.

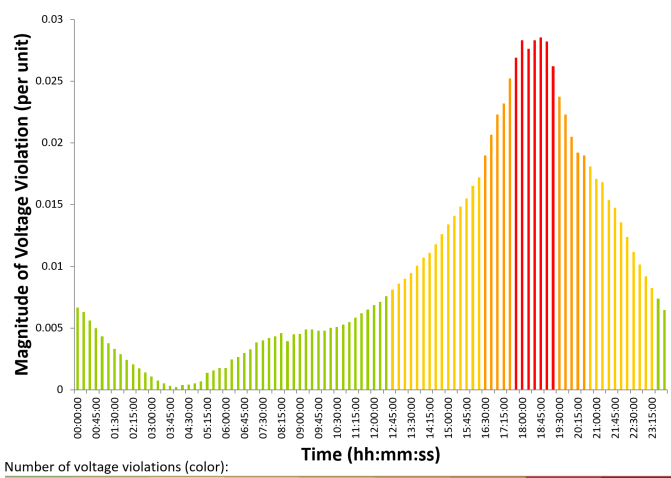

Fig. 5: Magnitude of the worst voltage violations over the day for the DC OPF with AC power flow dispatch

Findings for this study conclude that using more advanced dispatch methods may result in cost savings, less emissions, and no expected voltage constraint violations compared to the status quo of DC OPF with AC PF correction, typically used by grid-operators due to ensured feasibility and being less computationally intensive. This presents a case for a software upgrade to the grid, but also an opportunity for electric power decarbonization efforts [20].

V. Conclusion

Overall, the findings suggest that current dispatch methods implementing power flow corrections to ensure AC feasibility often rely on high carbon generation sources. The AC OPF (assuming to require no power flow correction), results in the lowest carbon emissions and price. This is because the AC OPF selected generation resources that have low prices, but these sources in the considered test case, also have low carbon emissions associated with energy production. However, the AC OPF comes at a high computational expense and nonconvexity.

Whereas, the traditional approach of using DC OPF with a AC power flow correction, results in the highest carbon emissions and price in the chosen test network. A middle ground is found using a proposed alternative method, where the DC OPF is run but the correction seeks the nearest feasible AC point. This is distinct from using an AC power flow correction, where generator outputs are fixed, because the generator outputs are adjusted in order to meet AC feasibility. The proposed method significantly reduced both cost and carbon emissions – highlighting the impact of the slack generator on both price and carbon emissions. Thus, if using DC OPF it is suggested to distribute the slack throughout the network rather than relying on a single slack bus – given the nature of slack generators being quick ramping, high cost, and high carbon generation sources.

Previous works, including [21], which uses a non-linear optimization problem to select the best slack bus, suggest that having a single slack bus in a network minimizes power imbalance in load flow studies. Thus, future studies may consider CO2 emissions in the selection of a slack bus, and the impacts on dispatch methods. Further, future studies should consider a comparison of dispatch methods, feasibility, and computational intensity on realistic grid models. While this study did consider an alternative to traditional AC or DC dispatch methods, other methods that ensure AC feasibility could be considered to reduce computational cost for realistic grids [22]. Further, it is suggested to consider a more temporal approach, considering longer time spans and seasonal variations – particularly periods when the grid may be under stress. Temporal carbon coefficients from the NREL tool Cambium could then be used to better account for generation efficiency and losses, but also scenario-driven emissions due to climate change and policy to better understand the relationship between carbon emissions, price, and dispatch methods.

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