Co-ordinated Electric Vehicle Charging to Reduce Losses Without Network Impedances

Constance Crozier[1]*, Matthew Deakin[2], Thomas Morstyn[1], Malcolm McCulloch[1]

[1] *Department of Engineering Science, University of Oxford, Parks Road, Oxford, UK*
[2] *School of Engineering, Newcastle University, Queen Victoria Road, Newcastle-upon-Tyne, UK*
* *E-mail: constance.crozier@eng.ox.ac.uk*

Abstract: This paper proposes a method of coordinating electric vehicle charging to reduce losses in a distribution system, using only knowledge of the phase that each charger is connected to. Reducing network losses cuts costs and can be achieved through demand response mechanisms. However, directly minimising losses requires accurate values of the line impedances, which can be difficult to obtain. Flattening load over time and balancing load across phases have both been proposed as alternate solutions which indirectly reduce losses. Here, the practical differences between load flattening and explicitly minimising losses are quantified using simulations of residential charging in European style, 3-phase distribution networks. Then, a new smart charging strategy, which incorporates phase balancing as a secondary objective to load flattening, is proposed. This requires only the knowledge of the phase that each load is on and achieves 30-70% of the potential reduction in losses.

Nomenclature

Variables

i	Node currents (A)
κ	Uniform demand applied to all nodes (kW)
L	Total energy lost (Wh)
p	Real power injected (W)
q	Reactive power injected (VAR)
s	Power injections into the nodes (VA)
v	Node voltages (V)
x	Vehicle load (W)

Parameters

a	Linear power flow model bias (V)
E	Charging energy requirement (Wh)
h	Household load (W)
$\mathbf{M}_y$	Linear power flow model load weights (VW^{-1})
N_b	Total number of buses on the network
N_h	Total number of households
P_{max}	Maximum possible charging power (W)
$\underline{t}$	Time interval in which the vehicle arrives
$\overline{t}$	Time interval in which the vehicle departs
T	Total number of time intervals
Y	Admittance matrix (Ω^{-1})
α	Overall power factor
Δt	Size of each time interval (mins)
η_c	Charging efficiency
λ	Weighting in multi-objective optimisation

Sets

$\mathcal{H}_x$	The set of households on phase x

Indicators

ϵ_V	Relative voltage error

Notation

$.^{(j)}$	Element belonging to vehicle j
$.^{(k)}$	Element at node k
$._L$	Array with the slack bus removed
$._0$	Array relating to the slack bus
$._t$	Element at time t
$.^*$	Complex conjugate
$Re\{x\}$	Real component of x
$f()$	Primary objective function
$g()$	Secondary objective function

1 Introduction

This paper develops a method of charging electric vehicles in distribution networks that reduces resistive losses without requiring a full model of the network.

Electric vehicles (EVs) are rapidly gaining popularity in high-income countries, driven by government incentives and falling battery prices. In the UK, as of Q1 2020 there are 288,000 plug in electric vehicles on the roads [1], and this number is forecast to rise to 36 million by 2040 [2]. As at least 50% of vehicles are parked at home at any time, charging at home offers the most convenience and flexibility [3]. However, mass charging of EVs at home would significantly increase loading on distribution systems [4], and appropriate action is required to ensure that this will not disrupt system operation.

Low voltage distribution networks currently account for 29% of electrical losses in the UK power system [5], and EV charging is expected to increase this share [6]. Losses not only waste electricity, but generate heat in the cables and transformer – leading to degradation of the cables' insulation [7], and a reduction in the transformer's lifespan [8]. Therefore, minimising system losses can reduce both operating and fixed costs. Loss minimisation in distribution networks is a mature research topic; solutions have focused on network reconfiguration, placement of capacitors, and adding distributed generation [9, 10]. For example, in [11] they optimise the placement of new switches in a LV network to minimise losses. However, these methods incur large costs, and flexibility of EV charging demand means that losses can instead be reduced by shifting the demand in time, or *smart charging*. This method would be preferable for

network operators due to the large costs associated with network reinforcements.

Explicitly minimising losses with EV charging is the theoretically optimal solution for distribution network operators (DNOs). However, it requires a complete model of the network impedances, which can be difficult to calculate, and would have to be updated every time the network is reconfigured. Additionally, the problem is non-polynomial hard and non-convex, meaning the computation time of the problem scales poorly with the number of loads and time resolution [12]. This is largely because the non-linear power flow equations result in non-convex optimization formulations. Nevertheless, several studies have proposed smart charging algorithms that minimise losses using the full equations [13–15]. However, the computational burden necessitates simplifications in the problem set up; [13] uses a time resolution of 1 hour and treats EVs as homogenous, [14] assumes a flat voltage profile, and [15] only considers reactive power for reducing losses. Further examples of non-convex loss minimisation can be found in micro-grid optimal power flow problems (e.g. [16, 17]); these are also limited to small networks at coarse time resolutions. At low time resolutions it is not possible to estimate losses accurately; using a half-hourly time resolution underestimates the losses by 7% [18]. Therefore, loss minimisation algorithms which use time resolutions of 1 hour will not necessarily find the minimum possible loss scenario.

Alternatively, it is possible to formulate approximate loss minimisation problems that can be solved in polynomial time. Typically, these involve simplification of the power flow equations, in order to remove the quadratic constraint. For example [19, 20] use a DC approximate power flow in order to minimise losses in a micro-grid. However, [19] uses an iterative approach, calculating the full power flow equations in each iteration to reduce error, and [20] uses locational marginal prices which are calculated based on AC power flows. This means that neither formulation can be solved in polynomial time, and therefore they scale poorly with the number of buses and time periods. Local problems that do not require solving the power flow equations can also be formulated (e.g. [21]) – although there is no guarantee that the global optima is found. These approximate methods still require the full network model.

Due to the technical challenges of minimising losses, smart charging research has focused on other objectives that indirectly reduce losses. A common approach is to flatten the sum of all of the applied loads on a network (e.g. [22–24]). Flattening load is often assumed to be equivalent to minimising losses (e.g. [25]), and distribution network operators (DNOs) are predominately focusing on shifting load to off-peak times in order to reduce losses [5]. Flattening load will minimise peak demand, protecting the transformer (since the power flow at the transformer depends on the sum of the downstream loads), but not necessarily the lines that are distant from the substation. Losses vary quadratically with current, so higher peak loads result in larger losses, and in [26] it was found that flattening EV charging load reduced losses by 20% compared to uncontrolled charging. However, minimising losses is only strictly equivalent to flattening load if all loads are drawn from a single bus on the network [27]. The main advantages of load flattening are its computational simplicity, and small number of required parameters. The problem can be formulated as a convex problem which can be solved in polynomial time [23], and no information about the network structure is required.

In 3-phase networks, imbalance between the phases increases losses, so reducing phase imbalance has also been suggested as a smart charging objective. Phase imbalance is caused by both systemic imbalance in the network, and imbalance in the applied load [28]. While the former can not be easily changed, the latter can be reduced by shifting demand in time. In the case of 3-phase smart chargers, load can be shifted in real-time from one phase to another (e.g. [29, 30]). However, the majority of domestic EV chargers are connected to a single phase, so it is necessary to co-ordinate the fleet of vehicles. The resulting problem is non-convex and requires a full network model (e.g. [31, 32]). However, as the assigned phases of the households and vehicle loads are fixed, imbalance can be reduced to an extent heuristically, by taking into account the phase that each load is on. While imbalance between phases results in additional losses, the minimum losses do not occur at the minimum phase imbalance. This is because phase imbalance does not take account of the size of the load, so well-balanced large loads result in high losses. For this reason, direct minimisation phase imbalance using smart charging is not considered in this paper.

To the authors' knowledge, no existing smart charging formulations reduce losses beyond flattening load without requiring the full network model. The novel approach presented in this paper uses the assigned phase of loads to achieve superior performance with only a small increase in computational complexity. Unlike network impedances, assigned phases are relatively easy for DNOs to determine, and would only need to be found once.

In this paper, a smart charging method is proposed which incorporates phase balancing into the standard load flattening algorithm. The performance of this scheme, load flattening, and explicit loss minimising are analysed for a range of networks and loading conditions. This is achieved using a stochastic modelling framework based on real data and convex formulations for the problems. Given forecasts for loads, this method can be used by an aggregator on behalf of a distribution system operator to optimise EV charging to reduce losses, either on a day-ahead basis or, using a sliding window, in real-time.

The contributions of this paper can be summarised as follows: First, a smart charging method is presented which reduces the losses in a network beyond that which is achieved by flattening load, without requiring the network impedances. This distinguishes it from the existing formulations in the literature, which either indirectly reduce losses by flattening load, or explicitly minimise losses using the network impedances. Second, the action of the proposed method is compared to load flattening and loss minimisation in stochastic simulations. Both the effects of load flattening and loss minimisation have been previously investigated in residential networks, but they have not been directly compared using computationally equivalent formulations. The comparison here uses a consistent set of modelling assumptions and time resolution for all three formulations, and investigates the losses, peak load, and computation time in each case. Third, a sensitivity analysis is conducted to understand the dependance of the results on network, season, and EV population.

The results in this paper focuses on European style 3-phase distribution networks and use data specific to the UK. However, the framework is applicable to other styles of network and, with appropriate data, results could be replicated for other areas. Only uni-directional charging is considered; further work would be required to adapt the algorithms for bi-directional charging.

The remainder of this paper is structured as follows. Section 2 details the modelling framework used to construct realistic EV charging simulations. The convex smart charging algorithms are described in Sections 3 and 4. Results are presented in Section 5 and Section 6 concludes the paper.

2 Modelling Framework

In order to quantify the effectiveness of the proposed strategy at reducing losses, simulations representative of residential EV charging are required. Load at this level of aggregation is highly stochastic, so it is important to incorporate variability of individual loads into results. In this section, the modelling framework used to construct simulations representative of residential EV charging is detailed.

Monte Carlo simulations are used to capture load stochasticity. Historic data is used for both the household demand profile and the EV charging demand. The data are segmented into time periods consistent with the length of the simulation. For each run of the simulation, vehicles and household data are randomly added to the network. Individual loads are selected from a set of scenarios using a random number generated from a uniform distribution. Once selected, the load is removed from the distribution to avoid repetition of loads in a single simulation run.

Here a 100% penetration of EVs is defined to mean that there is one EV at each household. Note that vehicles are assigned to households regardless of whether they were used in the period in

question. This means that some of the vehicles may have zero energy demand over the simulation time range. The system losses and peak load are then evaluated for several scenarios: without the EVs, with uncontrolled charging of EVs, and with each smart charging scheme. Running this simulation once produces a single deterministic value; a distribution of values is obtained by repeating the simulation, with loads randomly chosen and positioned in each simulation instance.

The uncontrolled charging scenario is taken to mean that the vehicles charge as they were observed to. For the smart charging scenarios, EVs are required to receive the amount of charge that they were observed to have demanded, between the time they were plugged in and the time they were next used. Note that this means that if a vehicle is unplugged before it is fully charged, then it is considered to have no flexibility, and the smart charging profile will be identical to the uncontrolled profile.

Here it is assumed that an aggregator has direct control of the associated EV chargers, such that they can choose the charging profile that vehicles follow. The availability and energy requirements of the individual vehicles are respected, however the aggregator can decide how these requirements will be met. One alternative solution is that the aggregator provides a price signal to charge points which then optimise their own charging to minimise cost. The proposed method focuses on the objective rather than the problem constraints, so with further work a decentralised version of the methodology could be developed that uses price signals to control vehicles.

3 Loss minimising algorithm

In this section, the linearisation of the power flow equations described in [33] is used to formulate loss minimisation as a convex problem.

3.1 Linear 3-phase power flow

In the case of a three phase, unbalanced distribution network with wye-connected loads and N_b 3-phase buses, we can write down the power flow equations as

$$\mathbf{s} = \text{diag}(\mathbf{v})\mathbf{i}^*, \tag{1a}$$

$$\mathbf{i} = \mathbf{Y}\mathbf{v}, \tag{1b}$$

where $\mathbf{s}, \mathbf{i} \in \mathbb{C}^{3N_b}$ are are vectors of the complex power and current injections at each bus, $\mathbf{Y} \in \mathbb{C}^{3N_b \times 3N_b}$ is the network admittance matrix and $\mathbf{v} \in \mathbb{C}^{3N_b}$ is the vector of bus voltages. The admittance matrix and voltages can be decomposed as:

$$\mathbf{Y} = \begin{bmatrix} \mathbf{Y}_{00} & \mathbf{Y}_{0L} \\ \mathbf{Y}_{0L} & \mathbf{Y}_{LL} \end{bmatrix} \quad \mathbf{v} = \begin{bmatrix} \mathbf{v}_0 \\ \mathbf{v}_L \end{bmatrix}, \tag{2}$$

where $\mathbf{Y}_{00} \in \mathbb{C}^{3 \times 3}, \mathbf{v}_0 \in \mathbb{C}^3$ describe the slack bus. The linearisation described in [33] is followed, such that the bus voltages are approximated as:

$$\mathbf{v} = \mathbf{M_y} \begin{bmatrix} \mathbf{p} \\ \mathbf{q} \end{bmatrix} + \mathbf{a}, \tag{3}$$

where $\mathbf{p}, \mathbf{q}$ contain the real and reactive load injections respectively, and

$$\mathbf{M_y} = \begin{bmatrix} \mathbf{Y}_{LL}^{-1}\text{diag}(\bar{\mathbf{v}}_L)^{-1} & -j\mathbf{Y}_{LL}^{-1}\text{diag}(\bar{\mathbf{v}}_L)^{-1} \end{bmatrix} \tag{4a}$$

$$\mathbf{a} = -\mathbf{Y}_{LL}^{-1}\mathbf{Y}_{L0}\mathbf{v}_0, \tag{4b}$$

which is calculated around a known power flow solution $\bar{\mathbf{v}}$ (the linearisation point). Given that there will only be loads placed on a subset of the buses on the network, we can remove columns from $\mathbf{M_y}$ which correspond to buses without loads on. This means that $\mathbf{M_y} \in \mathbb{C}^{3N_b \times 2N_h}$, where N_h is the number of households (or applied loads) on the network.
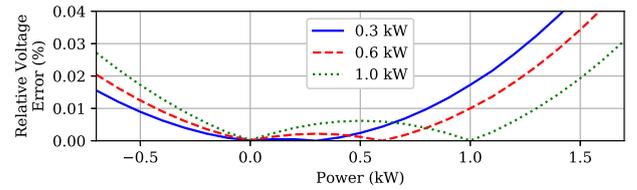

Fig. 1: Relative voltage error ϵ_V for three linearisations $\mathbf{M_{\bar{s}}}$, with a uniform demand κ assigned to each residential load (with a fixed power factor). The small error shows the linear models have good accuracy.

To validate the linear models (prior to optimisation), we consider a *relative voltage error*, ϵ_V, given by

$$\epsilon_V = \frac{||\mathbf{v} - \bar{\mathbf{v}}||_2}{||\bar{\mathbf{v}}||_2}. \tag{5}$$

Power flow solutions and the admittance matrix are both obtained using OpenDSS [34]. We study the error ϵ_V for three linearisations: one linearisation $\mathbf{M_{0.3}}$ with all loads at 0.3 kW, then $\mathbf{M_{0.6}}$, $\mathbf{M_{1.0}}$ likewise at 0.6 kW and 1.0 kW respectively. For each of these models, a uniform demand κ is applied to all loads, and the error calculated by comparison to the true power flow solution (see Fig. 1). There is zero error at the linearisation and no-load points, as expected [33], and less than 0.02% error within ±50% of the linearisation point. Errors in voltage can be mapped (approximately quadratically) to errors in the final losses model. In Section 5.1 it is demonstrated that the charging regime can affect the total resistive losses by more than 30%. Given that this difference is several orders of magnitude larger than the error, it was concluded that this level of accuracy is acceptable for the applications in this paper. The mean load of the smart meter data was approximately 0.6 kW, and so the model $\mathbf{M} = \mathbf{M_{0.6}}$ is used subsequently.

3.2 Real losses model

The complex power injection at a given node k is given by:

$$s^{(k)} = v^{(k)}i^{*(k)} = v^{(k)}[\mathbf{Y}\mathbf{v}]^{*(k)}, \tag{6}$$

where $v^{(k)}$ is the node voltage, $i^{(k)}$ is the current injection at the node. The losses can then be written as:

$$\sum_{k=1}^{N_b} Re\{s^{(k)}\} = Re\{\mathbf{v}^\top \mathbf{Y}^* \mathbf{v}^*\}. \tag{7}$$

In order to convert this expression from voltages to applied loads we need to substitute in (3). In this case, only one type of load is considered (EV chargers) so it is reasonable to assume a fixed power factor, meaning the reactive applied load can be expressed as:

$$\mathbf{q} = \alpha\mathbf{p}, \tag{8}$$

where $\alpha \in \mathbb{R}$ is some constant. If each load had a different power factor, then α would be a diagonal matrix. Considering:

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_p & \mathbf{M}_q \end{bmatrix} \tag{9}$$

such that $\mathbf{M}_p$ contains the elements multiplied by real loads, and $\mathbf{M}_q$ the imaginary loads. Then substituting in (8), the linear power
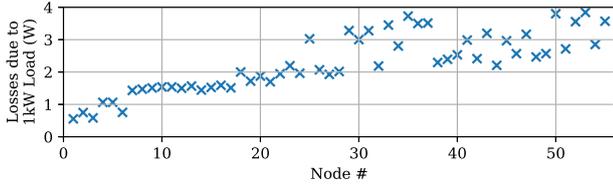
Fig. 2: The power lost when a single 1 kW load is applied to one node.

flow equation in (3) is reduced to:

$$\hat{\mathbf{M}}\mathbf{p} + \mathbf{a} \tag{10}$$

where:

$$\hat{\mathbf{M}} = \mathbf{M}_p + \alpha\mathbf{M}_q. \tag{11}$$

Finally, the losses can be expressed as a quadratic function of the applied real power:

$$\sum_{k=1}^{N_b} Re\{s^{(k)}\} = \mathbf{p}^\top \mathbf{\Lambda}\mathbf{p} + \boldsymbol{\gamma}^\top \mathbf{p} + c, \tag{12}$$

where:

$$\mathbf{\Lambda} = Re\{\hat{\mathbf{M}}^\top \mathbf{Y}^* \hat{\mathbf{M}}^*\}, \ \boldsymbol{\gamma} = Re\{2\mathbf{a}^\top \mathbf{Y}^* \hat{\mathbf{M}}^*\},$$
$$\text{and } c = Re\{\mathbf{a}^\top \mathbf{Y}^* \mathbf{a}^*\}. \tag{13}$$

This model allows the loss sensitivity of each node to be calculated. Figure 2 shows the estimated power lost when a single 1 kW load is placed on a given node in the IEEE European LV Network, and all other nodes are left unloaded. In general, the size of the losses is proportional to the electrical distance of the node from the substation. From the figure, it can be seen that the losses are 8 times higher when the load is at the bottom of the network than when it is at the top.

3.3 Optimisation problem

Consider a fleet of N vehicles over T discrete time intervals of duration Δt. The charging power of the vehicle at node j during time interval t is given by $x_t^{(j)}$. The vector $\mathbf{x}^{(j)} \in \mathbb{R}^T$ then represents the proposed charging profile of that vehicle over the whole time period, and $\mathbf{h}^{(j)}$ is the household's existing demand profile.

To minimise real power losses due to EV charging, the total energy lost over T time intervals must be considered:

$$\mathbf{L} = \Delta t \sum_{t=1}^{T} \sum_{k=1}^{N_b} Re\{s_t^{(k)}\}. \tag{14}$$

By considering $\hat{\mathbf{p}} = [\mathbf{p}_1 \dots \mathbf{p}_T]^\top$, the concatenation of the applied loads at all time instances, we can express:

$$\mathbf{L} = \Delta t\, \hat{\mathbf{p}}^\top \hat{\mathbf{\Lambda}}\hat{\mathbf{p}} + \Delta t\, \hat{\boldsymbol{\gamma}}^\top \hat{\mathbf{p}} + cT\Delta t, \tag{15}$$

where $\hat{\mathbf{\Lambda}} \in \mathbb{R}^{N_h T \times N_h T}$ is a block diagonal matrix of $\mathbf{\Lambda}$, and $\hat{\boldsymbol{\gamma}} \in \mathbb{R}^{N_h T}$ contains T concatenated copies of $\boldsymbol{\gamma}$. The real power load, $\hat{\mathbf{p}}$, can be decomposed into the uncontrollable household load, $\mathbf{h}$, and the controllable EV load, $\mathbf{x}$. The total losses can then be expressed as:

$$\mathbf{L} = \mathbf{x}^\top \hat{\mathbf{\Lambda}}\mathbf{x} + [(\hat{\mathbf{\Lambda}} + \hat{\mathbf{\Lambda}}^\top)\mathbf{h} + \hat{\boldsymbol{\gamma}}]^\top \mathbf{x} +$$
$$\mathbf{h}^\top \hat{\mathbf{\Lambda}}\mathbf{h} + \hat{\boldsymbol{\gamma}}^\top \mathbf{h} + cT\Delta t, \tag{16}$$

The individual EV energy requirements and limit of the chargers are encoded in the problem constraints. Every time a vehicle j is

plugged in a new set of constraints are generated, which are defined by: the time interval in which the vehicle arrives, $\underline{t}$, the time it is needed by, $\bar{t}$, and the energy it requires, $E^{(j)}$. Mathematically these constraints can be described as:

$$\eta_c \sum_{t=\underline{t}}^{\bar{t}} x_t^{(j)} \Delta t = E^{(j)}, \tag{17a}$$

$$0 \le x_t^{(j)} \le P_{max} \ \forall t \in [\,\underline{t}\,, \bar{t}\,], \tag{17b}$$

where η_c is the charging efficiency, such that (17a) ensures that the vehicle has received the right amount of energy before it is required, and (17b) limits the charging power at every time to be non-negative and below a maximum value. Finally, the optimisation can be described as:

$$\underset{\mathbf{x}}{\text{minimise}} \quad (16) \qquad \text{subject to} \quad (17). $$

which takes the form of a quadratic program (QP). This is convex providing the matrix of the quadratic component is positive definite[35], and can be solved using standard software packages, such as *cvxopt*[36].

4 Proposed algorithm

In this section, the proposed smart charging methodology is explained. First the standard load flattening optimisation problem is described. Then, incorporating phase balancing as a secondary objective is proposed.

4.1 Basic load flattening formulation

Flattening the load on a network is equivalent to minimising the 2-norm of the total demand. The 2-norm is given by the absolute square sum of the components at each time. Therefore, taking the same variables as in Section 3.3, the load flattening objective can be described as:

$$f(\mathbf{x}) = ||\sum_{j=1}^{N_h}(\mathbf{h} + \mathbf{x})^{(j)}||_2^2. \tag{18}$$

The individual constraints are unchanged, as they describe the charger and vehicle requirements, so the final problem is:

$$\underset{\mathbf{x}}{\text{minimise}} \quad (18) \qquad \text{subject to} \quad (17), $$

which also takes the form of a QP.

4.2 Phase Balancing for Loss Reduction

The load flattening problem is ill-posed, as a unique solution does not exist. This is because all vehicles are treated as homogenous, meaning that charging can be shifted from one vehicle to another without affecting the objective function. It follows that there are a set of solutions which flatten load optimally, and the solver will just pick the one closest to its starting point – typically solvers initialise all variables as equal, so the resulting profiles tend to be slow and flat. Tikhonov regularization is a method of creating a unique solution that is preferable in some way. A second function is added to the objective, weighted by a very small number λ, such that the function only becomes significant once the initial optima has been found.

In this case, it is desirable to select a load flattening solution which results in lower losses. Phase imbalance is one cause of losses, and can be quantified using the ratio $|I_2|/|I_1|$ [37], where $I_{0,1,2}$ are the

zero, positive, and negative sequence currents. These are calculated by:

$$\begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix}, \qquad (19)$$

where $a = e^{j\frac{2\pi}{3}}$, and $I_{A,B,C}$ are the currents in phases A, B and C [38]. Minimising this ratio across the network is a non-convex problem and requires the full network model to be known. However, the branch currents are driven by the applied loads, so it follows that the average phase imbalance in the network could be reduced by balancing the applied load across the phases. It is not normally possible to alter the phase that a household or vehicle is drawing power from, since households are often connected to a single phase. However, it is possible to co-ordinate the individual charging of the electric vehicles relative to one another, so that the total load on the network is relatively balanced. Therefore, here the following convex objective is considered:

$$\begin{aligned} g(\mathbf{x}) = \|\sum_{j \in \mathcal{H}_A} (\mathbf{h}+\mathbf{x})^{(j)} - \sum_{j \in \mathcal{H}_B} (\mathbf{h}+\mathbf{x})^{(j)}\|_2^2 + \\ \|\sum_{j \in \mathcal{H}_A} (\mathbf{h}+\mathbf{x})^{(j)} - \sum_{j \in \mathcal{H}_C} (\mathbf{h}+\mathbf{x})^{(j)}\|_2^2 + \\ \|\sum_{j \in \mathcal{H}_B} (\mathbf{h}+\mathbf{x})^{(j)} - \sum_{j \in \mathcal{H}_C} (\mathbf{h}+\mathbf{x})^{(j)}\|_2^2, \end{aligned} \qquad (20)$$

where the set $\mathcal{H}_i$ contains the households in the networks on phase i, such that each household belongs to exactly one set. While calculating losses requires all of the network impedance to be known, $g(\mathbf{x})$ requires only each load's phase. This is easier for DNOs to determine, and would likely only need to be found once (unlike network topology, which regularly changes as a result of network reconfiguration). It is shown in Section 5 that incorporating this function as a secondary objective to the load flattening problem has the effect of reducing $|I_2|/|I_1|$ across the network. The proposed optimisation can be described as:

$$\underset{\mathbf{x}}{\text{minimise}} \quad f(\mathbf{x}) + \lambda\, g(\mathbf{x}) \qquad \text{subject to} \quad (17)$$

where $\lambda \ll 1$, such that the function g only becomes significant in the optimisation search gradients once the minima of f has been reached. The resulting problem remains a QP.

5 Results & Discussion

In this section the results of the proposed smart charging objectives in realistic simulations are compared. First, the data sources used to formulate the simulations are described. Then, a detailed analysis of the proposed method, load flattening, and loss minimising on the IEEE European Low Voltage Test Feeder is presented. Finally, a sensitivity analysis is conducted, testing the dependance of the results to season, EV population, and network structure.

5.1 Data sources

Realistic charging simulations require data sources for both household electricity and EV charging demand. Here 1 minute resolution smart meter data from a UK trial were used for load profiles [39]. These were from 280 households in the North of England, which were monitored from November 2012 to July 2014. A single day from the dataset was chosen for a single simulation, so that the likely correlation between households' loads is captured. A dependancy on season and weather means that demand of nearby households on the same day are likely to be highly correlated. If this correlation is not taken into account the peak demand on the network may be underestimated.

For EV charging demand, we use data from a UK trial which monitored the charging of 224 Nissan Leafs [40]. Charging start and end times were recorded to the nearest minute, and the starting and final charge of the vehicle to the nearest 2 kWh. Each charging event was translated into a set of constraints (17a) and (17b), where $E^{(j)}$ is the total amount of energy the vehicle uses, $\underline{t}$ is the time that the vehicle arrives and is plugged in and $\bar{t}$ is the time that the vehicle departs. If arrival and departure times are uncertain, a sliding window could be incorporated to adjust for errors in forecasting.

For each vehicle, up to 18 months of data are recorded, but in this paper a reduced time range of 9 months is considered, where the overlap between vehicles' recording periods was largest. Unfortunately, it was not possible to select the same day or location for both as the household and EV trials did not overlap. The extent to which vehicle use and electricity demand are related is uncertain, however this may result in the peak loading on the network being underestimated.

The losses resulting from additional load will be dependant on the distribution network characteristics. In the UK, distribution networks are typically 3-phase, and a mix of radial and meshed styles. In this paper, a selection of the 3-phase test feeders from [41] are used, which are based on real networks in the UK. The first of these is the 55 household *IEEE European Low Voltage Test Feeder*, which was used for the bulk of the analysis. However, to gain some perspective on the variation between networks, 8 other systems were chosen. All test systems considered were 3-phase and operated at a frequency of 50 Hz with a base voltage of 230 V, consistent with the UK power network.

5.2 IEEE European Low Voltage Test Feeder

Monte Carlo simulations were constructed on the IEEE European Low Voltage Test Feeder using the methodology described in Section 2. Five scenarios were considered:

- No EVs – only the household loads were included.
- Uncontrolled charging – EVs charge as observed.
- Loss minimising – EVs charge according to Section 3.
- Load flattening – EVs charge according to Section 4.1.
- Load flattening with phase balancing – EVs charge according to Section 4.2.

All optimisation problems were formulated at 1 minute resolution, with a maximum vehicle charging power of 3.5 kW – which is the rated power of typical domestic charging points in the UK. The total load profiles resulting from the scenarios are shown in Fig. 3. Note that load flattening with phase balancing is not included, as the load profile is identical to load flattening. On average, uncontrolled charging of EVs doubled the existing peak demand and significantly increased the variability in demand. Flattening load mitigated the increase in peak demand, while minimising losses resulted in a 20% increase. Whether or not this is acceptable will depend on the rating of the distribution transformer.

The losses experienced in each scenario are shown in Fig. 4, where the blue lines show the median values, the boxes cover 50% of the values, and the whiskers show the range. The addition of the EV charging significantly increased the network losses, regardless of charging scheme; however all smart charging strategies reduced losses compared to the uncontrolled charging case. As expected, the losses were lowest when they were explicitly minimised. Losses that occur in the other smart charging scenarios, but not in explicit loss minimisation, could in theory be avoided. The effectiveness of load flattening and the proposed load flattening with phase balancing strategies can be compared using the amount of theoretically avoidable losses that are realised. The addition of phase balancing reduced avoidable losses by an average of 54% compared to the load flattening cases. This means that the addition of the secondary objective achieves more than half the benefit that explicitly minimising losses would provide, without requiring the network topology and impedance information.
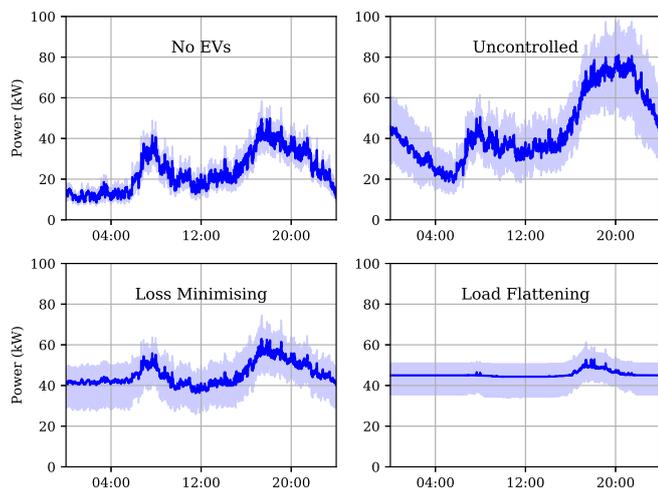
Fig. 3: The total load on the 55 bus network, without EV charging and under three different charging scenarios. The solid lines show the median load over the simulations and the shaded area covers the 90% confidence interval.
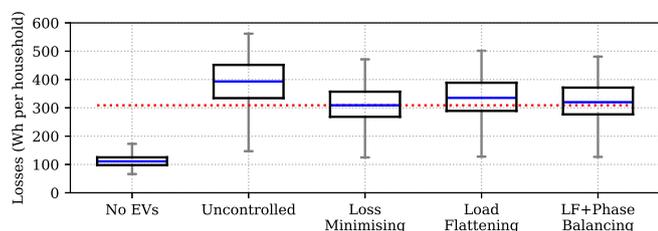

Fig. 4: The energy losses per household experienced under the five scenarios. The blue line shows the median, the box covers the interquartile range, and the whiskers cover the total range. The dotted line aids comparison against the loss minimising median.

Fig. 5: The test network under each EV charging regime, where the line colours show the average phase imbalance. Single phase lines are grey.

Fig. 6: The average losses per length in each branch of the network under load flattening + phase balancing when compared to loss minimising.

Load Flattening	Load Flattening + Phase Balancing	Approx Loss Minimisation	Full Loss Minimisation
6.0s	12.3s	8.7s	X

Table 1 The average simulation time for the smart charging solution to be calculated for 1 day of charging at 1 min resolution on the IEEE European Low Voltage Test Feeder.

Figure 5 shows the average phase imbalance throughout the simulation in all 3-phase lines, for each EV charging scenario. The values are capped at 10% and 50%, and the coloured markers show the phase that each household is on. Under all scenarios the phase imbalance is worse further down the network, which is unsurprising as the systemic imbalance will be greater at this level of aggregation. All the smart charging schemes exacerbated the imbalance at the bottom of the network compared to uncontrolled charging, potentially because in the uncontrolled case the EVs broadly follow similar charging profiles (predominantly overnight). This demonstrates the fact that phase imbalance does not take account of the size of the load, so would not necessarily reduce losses if used as the primary objective. The load flattening with phase balancing has the smallest phase imbalance at the top of the network. This is unsurprising, as (20) considers the imbalance of all of loads summed – which is analogous to the load at this point in the network.

Loss minimising results in a worse phase imbalance at the top of the network compared to load flattening with phase balancing. This, perhaps surprising, result reiterates the point that phase imbalance alone can not be used as a measure of losses. However, more clarity can be gained by visualising the location of the losses in the network for these two schemes. Figure 6 shows the difference in average losses per meter over the day between phase balancing and loss minimising, for each line in the network. At the top of the network, where the load is flatter and better balanced, the losses are lower in the phase balancing case, while further down the network the losses are lower in the loss minimising case. Therefore, while the losses are largest at the top of the network, the total losses are minimised by flattening the load elsewhere in the network.

While all of the smart charging formulations are quadratic programs, and therefore can be solved in polynomial time by standard solvers. The computation time will vary according to the sparsity of
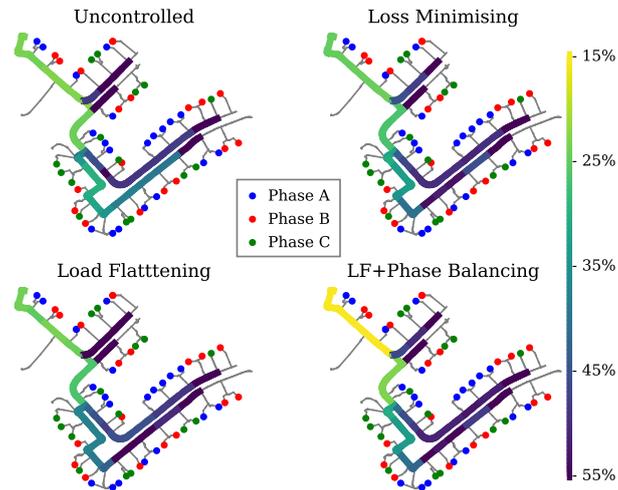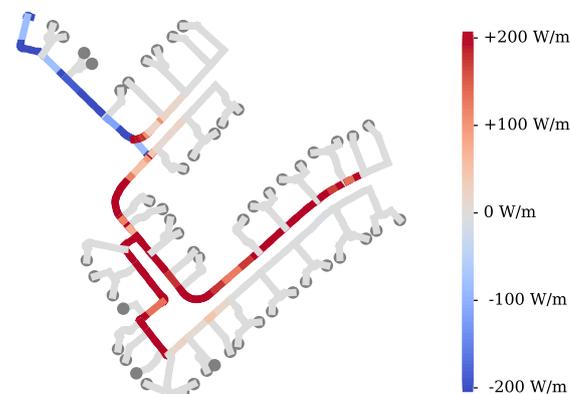
the quadratic matrix. The average calculation time for each smart charging solution is shown in Table 1. All optimisation strategies were carried out using cvxopt in Python on a 2.3 GHz Intel Core i5 with 8 GB of memory. The load flattening with phase balancing strategy has a higher computational complexity, due to the quadratic matrix in the objective having a larger number of off-diagonal terms. However, the main benefit of the proposed algorithm over approximation loss minimisation is not requiring the network impedances. If accurate impedance information is available it may be more beneficial to use the loss minimisation strategy. It is worth noting that the loss minimisation results are all using the approximation loss minimising algorithm from Section 4.1 – full loss minimisation is non-convex, and therefore at this time resolutional is not tractable on this simulation platform.
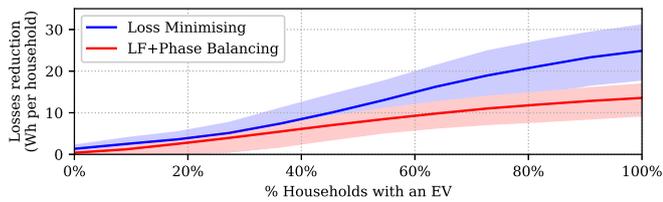
Fig. 7: The reduction in losses, per household with an EV, achieved by minimising losses rather than flattening load, against EV penetration. The solid line is the median, and the shaded area covers the inter-quartile range.
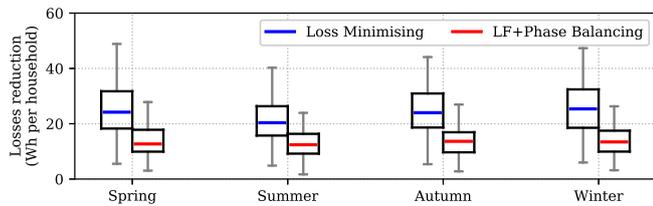

Fig. 8: The reduction in losses per household achieved by each scheme compared to flattening load, for various seasons. The thick lines shows the medians, the box covers 50% of the values, and the lines the total range.

5.3 Sensitivity Analysis

This section investigates the dependance of the results from Section 5.1 to various modelling parameters.

5.3.1 Sensitivity to EV Population: Thus far, it has been assumed that there is one EV at each household. However, smart charging is likely to be implemented before this penetration level is reached. It is therefore important to consider how the difference between these algorithms changes with lower levels of penetration. Figure 7 shows the additional loss reduction achieved by the more advanced smart charging algorithms when compared with load flattening. This is the difference between the losses in the smart charging scenarios compared to load flattening – note that the total losses increase quadratically with the total load on the system. The solid lines show the median value, and the shaded area covers the interquartile range. It can be seen that, regardless of penetration level, phase balancing achieves an average of 50% of the possible reduction in losses, and that there is an approximately linear relationship between EV population and loss reduction. This means that with fewer EVs on a network, the additional benefit of minimising losses is lower than that shown in Section 5.2. In other words, when the penetration of EVs is very low it may not be worth doing anything more advanced than flattening load. However, as the EV population increases, so do the potential loss reductions from incorporating phase balancing.

5.3.2 Sensitivity to Season: In the UK, heating and lighting contribute significantly to household electricity demand. Throughout the year there is a 12.7°C change in average temperature and an 8.8 hours change in daylight length. This means that the shape and size of household demand varies significantly with the time of year. To quantify the effect this has on the difference between the algorithms the simulation was repeated using load and vehicle data from different times of year. Figure 8 shows the additional reduction in losses achieved by the more advanced smart charging schemes, compared with load flattening for each of the seasons. There was minimal difference in the results, although slightly larger values were observed in the winter simulation (where the feeder was the most heavily loaded).

5.3.3 Sensitivity to Network Structure: Thus far, the results have focused on the IEEE European Low Voltage Test Feeder. However, network topology and the phase distribution of loads have a large effect on the losses in a distribution network. Therefore, the

		Losses (Wh per household)				
#	Loads	No EVs	Uncontr.	LM	LF	LF+PB
1	24	40	133	102	114	110
2	67	57	199	160	172	166
3	73	106	378	308	329	323
4	55	111	385	306	332	318
5	94	138	479	392	416	404
6	175	164	576	475	500	490
7	65	179	632	515	548	532
8	186	384	1344	1153	1171	1157
9	115	481	1664	1379	1442	1396

Table 2 The daily losses in each of the networks considered.

simulation described in Section 5.2 is repeated for 8 other feeders from [41]. The full results are displayed in Table 2. Figure 9 shows the additional loss reduction and associated increase in peak demand for the two algorithms, compared with flattening load, for each network. As expected, load flattening with phase balancing does not increase peak demand compared with load flattening. The networks are ordered by their losses per household before EV charging is added, and the number of households on the network is shown in brackets. Network 4 is the IEEE European Low Voltage Test Feeder; a mapping of the feeders to those in [41] is presented in the Appendix.
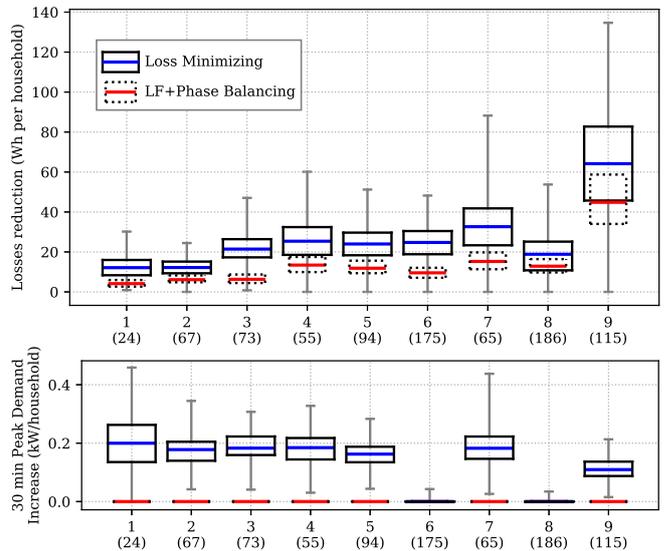

Fig. 9: The additional reduction in losses and associated increase in peak demand for each of the test networks. Both loss minimising and load flattening with phase balancing are shown, compared to the load flattening scenario.

For all of the feeders considered, flattening load resulted in a significant amount of avoidable losses. In general, this was larger for lossier feeders. For most of the feeders, the reduction in losses came at the expense of around a 0.2 kW increase in peak 30 minute demand per household. On average, balancing phase reduced between 30 and 70% of the avoidable losses, without an increase in peak demand.

5.4 Reward Allocation

There are two direct benefactors from reduced losses; the distribution network operators (DNOs) and the energy suppliers. DNOs benefit due to lower line loadings, and peak loads passing through the transformer. Reducing losses can defer required network reinforcements. Suppliers benefit because losses that are "in front of the meter" are not assigned to any particularly consumer, and are

therefore paid for by the supplier. However, in both cases these costs are inevitably passed on to consumers, so all bill-payers will indirectly benefit from reduced losses. The owners of the participating vehicles should also be directly remunerated by the aggregator in a manner proportionate to the value which they provided by delaying their charging. However, the exact allocation algorithm is outside the scope of this paper.

6 Conclusion

This paper proposed a method of charging electric vehicles in distribution networks which reduces resistive losses without requiring a full model of the network. By explicitly minimising losses rather than flattening load with a 100% EV penetration in the IEEE European Low Voltage Test Feeder, an additional 1W per household could be saved – although this came at the expense of a 0.2 kW per household increase in peak demand. The reduction in losses is due to two components: (1) prioritising load flattening on high impedance lines and (2) load balancing between phases. Based on this observation, a new smart charging strategy was proposed, which incorporates phase balancing as a secondary objective to load flattening. The proposed scheme achieved between 30–70% of this reduction in losses, without an increase in peak demand.

The additional benefit of loss minimising was found to be approximately linear with the number of EVs charging on the feeder, and did not vary significantly seasonally. The biggest sensitivity of the result was to the network structure; 9 feeders were considered and the average daily savings per household ranged 15 Wh to 70 Wh. The savings were approximately proportional to the losses in the network before EV charging was added. The fraction of losses saved was shown to have no significant relation to the number households on the network.

Further research will be required before the algorithms proposed in this work could be implemented in practice. First, a decentralised approximation of the proposed methodology may need to be developed. This would allow for random EV arrivals, and negates the data privacy concern from users having to transmit their availability. Secondly, if a decentralised scheme were developed, more complex modelling of the battery dynamics would be possible, due to the reduced complexity of optimising a single vehicle. Finally, further validation of the results could be performed once data become available from future trials which collect both EV and smart meter data from households.

Appendix

Description of the feeders utilised in the results section

The table below maps the feeders used in this analysis in Section 5.3.3 to those in [41].

#	1	2	3	4	5	6	7	8	9
Network	4	21	16	1	3	2	19	7	2
Feeder	1	3	2	1	1	1	3	4	4

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