Feasible Region-Based Heuristics for Optimal Transmission Switching

Constance Crozier, Kyri Baker, and Bridget Toomey

***Abstract*—In this paper, we develop a optimal transmission switching (OTS) heuristic based on DC optimal power flow (OPF) and assess the efficacy of the approach when implemented within AC OPF. Traditional formulations of the OTS problem can result in hundreds or thousands of binary variables for large networks, making the OTS problem challenging to solve on fast timescales even for relatively small networks. Here, we identify which constraints and therefore which variables are constraining the DC OPF feasible region, and rank them based on their impact on the cost function. We develop a heuristic algorithm which iteratively removes these constraints and solves a series of standard DC OPF problems. The heuristic is tested on a variety of PGlib networks and the results show that the algorithm can provide substantial cost decreases without having to solve any mixed integer programs. We provide additional insights about the OTS problem, including identifying scenarios outside congestion where OTS can prove useful. Lastly, the performance of the DC-based heuristic is shown when the line switching decisions are implemented within AC OPF.**

I. INTRODUCTION

Optimal transmission switching (OTS) is a tool that can allow grid operators to improve the economic efficiency and, in some cases, reliability of the system by connecting or disconnecting transmission lines [1]–[6]. OTS can also affect million-dollar decisions made in transmission capacity planning [7]. However, the OTS problem is computationally challenging even using the linear DC optimal power flow (OPF) model. This is because the problem contains binary variables corresponding to the line on/off decisions. OTS problems can take hours to solve using traditional computing platforms, or be completely intractable for large power systems. Solving OTS problems in short timescales is an ongoing challenge, as evidenced by the ARPA-E Grid Optimization Competition Challenge 2 [8]. In this paper we focus on the OTS problem only with regards to switching of transmission lines, however there are other works which consider switches at the distribution level for network reconfiguration (e.g. [9]–[11]).

The full OTS problem uses the AC power flow constraints, which are non-linear. However, mixed-integer non-linear optimization problems are extremely challenging to solve – there are no scalable open-source solvers, and the computational complexity of the problem means it may not be solved in the required time-horizon. Many formulations are instead based on the linearized DC power flow equations, which remove the quadratic and sinusoidal terms from the problem constraints. One common formulation for the line-switching constraints is using the big-M method to remove the line flow constraints of lines that are switched off [12]. The performance of the resulting problem depends heavily on the strength of the formulation, and finding the strongest upper bounds to be used is NP-hard [13]. Bound strengthening methods have been proposed for speeding up the formulation, e.g. for certain network structures [14] or using data-driven methods [15]. Some formulations use a convex quadratic relaxation of the AC power flow equations [16]–[18]. Others include a two-level approach with an AC feasibility check at the lower level [19], or incorporate additional constraints for the voltages and reactive power [20]. These methods likely produce more accurate power flow solutions, however they scale significantly slower than fully linearized approaches.

Even the DC formulation of the OTS problem is NP-hard, meaning that for large networks with many lines the problem solution time may be prohibitively long. A reduction in computational complexity of the OTS problem can be achieved by reducing the number of switchable lines to be considered. Towards addressing this issue, previous efforts to reduce the solution space or pre-screen for relevant lines of interest have been developed [6], [21]–[25]. In both [21] for the RTS-96 bus system, and [22] for network sizes up to 300 buses, power transfer distribution factors are utilized to heuristically determine possible line switching candidates. In [6], a ranking mechanism based on dual variables is proposed and demonstrated on a 662-bus network to expedite solving the OTS problem. Simplified versions of the OTS problem that maintain mixed-integer characteristics can also be solved in order to reduce computational complexity of the full OTS problem [26]. In [23] machine learning methods are investigated for line and algorithm selection in OTS. The line outage distribution factors are used to screen lines in [25], and shift factors are exploited in [24].

In [21], the authors noted that most economic benefits from OTS arise from switching just a small number of lines. We also aim to capitalize on this fact by identifying a subset of lines to consider in the OTS problem, eliminating possibly hundreds or thousands of binary variables and corresponding constraints. Similar to [6], this paper also uses a scheme based on dual variables and solving a sequence of simpler DC OPF problems. However, in this paper, we also leverage the relationships between those binding constraints and the variables they are restricting in the primal problem. In this way the algorithm attempts to distinguish which constraints, if

C. Crozier and K. Baker are with the Department of Civil, Environmental, and Architectural Engineering at the University of Colorado Boulder, Boulder, CO, USA. Emails: {constance.crozier, kyri.baker}@colorado.edu. B. Toomey is with Piction Health, Boston, MA, USA.

removed, may increase the size of the feasible region. We also do not require any additional tuning parameters or exogenous variables; line removals are based solely on the links between binding constraints and relevant variables.

Lastly, while DC OPF-based OTS has mainly been used to lower overall system costs by alleviating congestion in the network [1], [4], [27], [28], we show use cases where switching lines off can be useful outside of congestion management. The developed heuristic for determining lines to switch off is demonstrated on various Power Grid Lib - Optimal Power Flow (PGlib) test networks [29], and a "less greedy" version of the algorithm which has better performance but requires longer computation times is discussed. The results are promising, and in multiple cases the algorithm achieves the same solution as the commercial optimization package Gurobi without solving any mixed integer problems.

The proposed method can also be used to identify, or prescreen, a set of lines of interest that could be considered in a full OTS formulation. The DC formulation of the OTS problem used in this paper, and much of the literature, does not guarantee optimality in the nonlinear AC problem. This is because the linearization applied to the power flow equations mean that the solution will necessarily not satisfy the AC power flow equations [30]. However, methods for adjusting the solutions of DC OTS to meet AC feasibility have been proposed (e.g. [31]) and these could be coupled to the method proposed here to produce AC feasible results. The AC feasibility and cost benefits are additionally assessed in this paper to show the efficacy of the DC-based heuristic when the decisions are implemented using actual AC power flows.

II. Comparing DC OPF and Economic Dispatch

In this section we derive an alternative formulation of DC OPF that will facilitate the comparison between the feasible regions of the DC OPF problem and the economic dispatch problem. We discuss the system model, notation, and assumptions below.

- Assume a connected network with n buses and m lines. Define $\mathbf{p_g}$ as a vector of active power generation variables p_{gi} at bus i and $\mathbf{p_d}$ a vector of active power demands (which are constant and not variable) p_{di} at bus i.
- Define $n \times 1$ vector $\mathbf{x} = \boldsymbol{\theta}$ as the vector of voltage phase angles at all buses its corresponding upper and lower limits (element-wise) as $\underline{\mathbf{x}}$ and $\overline{\mathbf{x}}$, respectively.
- Define $n \times n$ matrix $\mathbf{B}$ as the bus admittance matrix.
- Define $m \times n$ matrix $\mathbf{F}$ to be the matrix describing the network connections, such that $\mathbf{Fx}$ computes the flow down each line. Also define the $m \times 1$ vector $\mathbf{f}$ to be the vector of flow limits f_k on each line k. For simplicity, assume a flow constraint exists for each line (if a flow constraint does not exist on line k, set $f_k > \sum \mathbf{p_d}$ such that this constraint is never active).
- Define $n \times 1$ vector $\mathbf{y} = \mathbf{p_g} - \mathbf{p_d}$ and its corresponding upper and lower limits (element-wise) as $\underline{\mathbf{y}}$ and $\overline{\mathbf{y}}$, respectively.

A. DC OPF with only inequalities

The DC OPF problem with line flow constraints can then be written as:

$$\min_{\mathbf{x},\mathbf{y}} \quad \mathbf{c}^T \mathbf{y} \tag{1a}$$
$$\text{s.t}: \ \mathbf{Bx} = \mathbf{y} \tag{1b}$$
$$-\mathbf{f} \leq \mathbf{Fx} \leq \mathbf{f} \tag{1c}$$
$$\underline{\mathbf{y}} \leq \mathbf{y} \leq \overline{\mathbf{y}} \tag{1d}$$
$$\underline{\mathbf{x}} \leq \mathbf{x} \leq \overline{\mathbf{x}} \tag{1e}$$

where $\mathbf{c}$ is a $n \times 1$ vector of cost parameters. Given that the vector $\mathbf{p_d}$ is not a variable, this objective is equivalent to the standard objective of minimizing a linear function of the generator costs, providing there is no more than one generator per bus. Constraint (1e) provides bounds on voltage angles that are seen in many formulations (e.g. [1]) but may or may not exist. Note that only generation is penalized in the standard DC OPF formulation, and so any bus that only contains demand will have a corresponding cost of zero. Using the relationship $\mathbf{y} = \mathbf{Bx}$[1], (1) can be rewritten in terms of voltage angles $\mathbf{x}$:

$$\min_{\mathbf{x}} \quad \mathbf{c}^T \mathbf{Bx} \tag{2a}$$
$$\text{s.t}: \ -\mathbf{f} \leq \mathbf{Fx} \leq \mathbf{f} \tag{2b}$$
$$\underline{\mathbf{y}} \leq \mathbf{Bx} \leq \overline{\mathbf{y}} \tag{2c}$$
$$\underline{\mathbf{x}} \leq \mathbf{x} \leq \overline{\mathbf{x}} \tag{2d}$$

Problem (2) is now only in terms of inequality constraints. By analyzing which of these constraints are binding at the optimal solution, we can attempt to identify a subset of lines which could be restricting the feasible region. Note that the formulation and same intuition explained here can be extended to consider both switching lines off as well as switching lines back on.

B. Alternate Economic Dispatch Formulation

Economic dispatch (ED) is a purely market optimization, which selects the cheapest generators irrespective of their position on the network. The ED problem can be written using the given notation as follows:

$$\min_{\mathbf{y}} \quad \mathbf{c}^T \mathbf{y} \tag{3a}$$
$$\text{s.t}: \ \sum_{i=1}^{n} y_i = 0, \tag{3b}$$
$$\underline{\mathbf{y}} \leq \mathbf{y} \leq \overline{\mathbf{y}} \tag{3c}$$

where constraint (3b) represents power balance. The optimal objective value of economic dispatch provides a lower bound on the DC OPF objective, because it uses the cheapest possible generation. If this solution is obtained from the DC OPF, then no congestion or "overconstrained" voltage angles that result

[1] Note that in order for $\mathbf{y} = \mathbf{Bx}$ to have a unique solution, $\mathbf{B}$ must be invertible; this can be achieved by fixing the slack bus angle to 0 and removing that respective row/column of the $\mathbf{B}$ matrix.

Constraints:	None	Line Limits	Line Limits w/OTS	θ Limits	θ Limits. w/OTS	Ref. Bus	Ref. Bus w/OTS
P_{g1}	500	400	500	251.5	500	Infeasible	500
P_{g2}	0	100	0	249.5	0	Infeasible	0
Ref. Bus	1	1	1	1	1	3	3
Angle	None	None	None	θ_2 +/-60°	θ_2 +/-60°	θ_2 +/-60°	θ_2 +/-60°
Line Limits	None	200 on Line 1-3	200 on Line 1-3	None	None	None	None
Line Status	All on	All on	Line 1-3 off	All on	Line 2-3 off	All on	Line 1-2 off
Cost	\$6000	\$7200	\$6000	\$6745.28	\$6000	Infeasible	\$6000

TABLE I
THREE-BUS RESULTS FOR DIFFERENT SCENARIOS WHERE LINE SWITCHING CAN BE USEFUL.

in price increases exist. However, if the DC OPF solution is not equal to the ED solution, it is possible (but not guaranteed) that OTS will be useful. Recall that, even without line flow constraints, the DC power flow equations provide relationships between voltage angles that may constrain the system and eliminate the ED solution from the feasible set. Where the solutions are not equal, we will use these constraints to determine which lines may be of value to switch.

Towards this, we form the ED problem in terms of voltage angles instead of active power generation variables by substituting $\mathbf{y} = \mathbf{Bx}$:

$$\min_{\mathbf{x}} \quad \mathbf{c}^T \mathbf{Bx} \tag{4a}$$

$$\text{s.t}: \quad \sum_{j=1}^{n} \sum_{i=1}^{n} b_{ij} x_j = 0, \tag{4b}$$

$$\underline{\mathbf{y}} \leq \mathbf{Bx} \leq \overline{\mathbf{y}} \tag{4c}$$

where b_{ij} is (i, j) entry of matrix B and x_j is the j-th entry of vector $\mathbf{x}$. It is now more evident that the DC OPF imposes additional constraints (2b), and optionally, (2d). Thus, our efforts should focus on which of these is restricting the feasible region of (2).

III. INSIGHTS INTO WHEN DC OTS IS USEFUL

Consider the following three-bus example for the situations discussed in this section. Binding constraints are shown in red in each figure to indicate the subset of the constraints that are restricting the feasible region from achieving the ED solution.

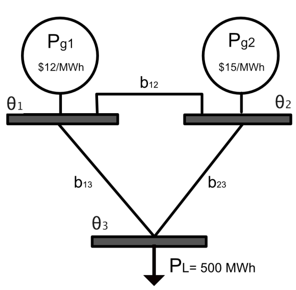

Fig. 1. Example 3-bus network.

In all of the following examples, assume that all line susceptances are equal ($b_{12} = b_{13} = b_{23} = -1$), there are no maximum generation outputs on the generators and the minimum generation output is 0 MWh. Three different scenarios where switching off three different lines in the network can benefit the overall system cost are discussed, and a summary of the results in the following sections are also shown in Table I.

A. When congestion is present

A canonical example of the value of OTS is in the presence of congestion. For this scenario, we do not consider upper/lower bounds on voltage angles, and $\theta_1 = 0°$ is the reference bus. Set the maximum flow limit along line 1-3 (in either direction) to have a magnitude of 200 MW, and notice that the system cost is \$7200, with an increase of \$1200 due to congestion. Then, remove line 1-3 from the network, as shown in Fig. 4. The system cost then becomes \$6000.

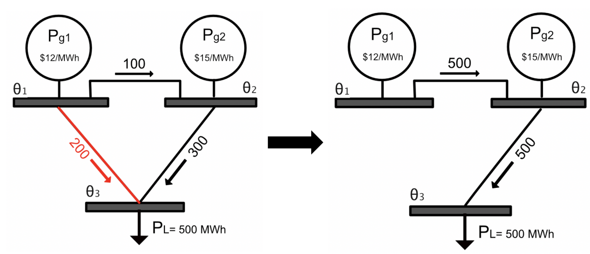

Fig. 2. Congested 3-bus network improves with OTS, with system costs going from \$7200 (left) to \$6000 (right).

B. When voltage angles are over-constrained

According to [32], the Braess's paradox occurs in optimal transmission switching problems due to the need for Kirchoff's voltage law (KVL) to hold around every loop in the grid (the sum of all voltage angle differences around a loop must be zero). This can "overconstrain" the values of some of the optimization variables, reducing the feasible region such that the achievable optimal solution worsens. Consider the three-bus network without any line flow constraints. Now, enforce $-\frac{\pi}{3} \leq \theta_2 \leq \frac{\pi}{3}$ which is a reasonable and not overly conservative bound. Due to the restriction on the angles ($\theta_1 = 0$, θ_2 bounded, and relationships between angles enforced by the power flow equations), the system cost becomes \$6745.28. Remove line 2-3 and notice that the system cost is now \$6000.

C. Choice of reference bus

In optimal power flow, the selection of a reference bus provides a reference for which all other bus angles are measured; however, a poor choice of reference bus can lead to

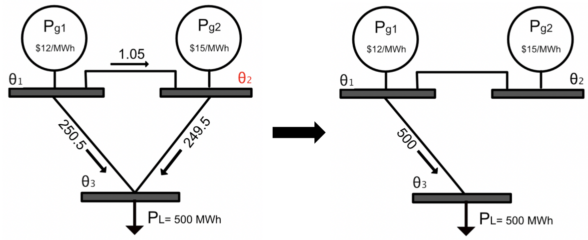

Fig. 3. Angle-constrained 3-bus network improves with OTS, with system costs going from \$6745 (left) to \$6000 (right).

overconstrained voltage angles. Consider the scenario mentioned in the previous subsection, where $-\frac{\pi}{3} \leq \theta_2 \leq \frac{\pi}{3}$. If bus 3 is chosen as the reference bus and $\theta_3 = 0°$, the problem actually becomes infeasible. However, if line 1-2 is removed, the problem becomes feasible, and the optimum cost function value of \$6000 is attained. Again, this is due to the overconstraining of voltage angles and relationships between angles. Note that if the constraint on θ_2 is removed, the DC OPF attains a feasible solution with a system cost of \$6000 even the reference bus chosen to be bus 3. This is a special case of the previous scenario of overconstraining a voltage angle.

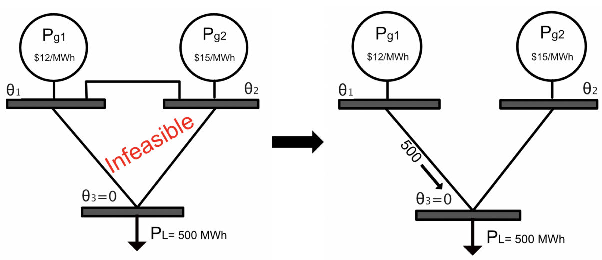

Fig. 4. Infeasible 3-bus network improves with OTS, achieving \$6000 (right).

D. Generalizations and cautions

Unfortunately, it is not always the case that the cost function can be improved by switching lines off. Switching some lines off, even if the network is still fully connected, may also result in infeasiblity (e.g. consider the case in Section III-A and attempt to disconnect line 2-3). In addition, it is important to note that although the example in Section III-A indicated that the optimal solution was to turn off the congested line, this is not always the case. Lastly, it is not normally observed that the choice of the reference bus or constraints on voltage angles will impact the feasible region of the problem on larger systems. The three-bus is useful for illustrating these broad concepts but in larger networks the angle relationships from these constraints are typically not as restrictive.

It is also important to note that we do not consider N-1 security in this paper, although this would be an important consideration in practice to avoid sacrificing reliability in order to improve cost [33], [34]. Additionally, it has been pointed out by some authors [3], [35] that lines switched off in a DC OPF formulation may not provide optimal switching choices under an AC OPF framework. Towards analyzing how this particular heuristic impacts the AC feasibility and cost benefits within AC OPF, we provide results on all of the considered test cases assessing the performance of the DC-based heuristic within a more realistic AC OPF environment.

IV. Narrowing down lines of interest

Using the intuition gleaned from the scenarios in the previous section and the formulations of ED and DC OPF in terms of bus angles only, we now design an algorithm to determine which lines, if switched off, may increase the size of the feasible region. Formulating the ED and DC OPF problems in terms of only voltage angles allows us to more easily determine which constraints restrict the feasible region in DC OPF.

A. Constraint analysis

The Lagrange multipliers corresponding to the constraints in the DC OPF and ED problems represent the sensitivity of violating that constraint with respect to a change in objective function value. With respect to the inequality constraints, the Lagrange multipliers represent whether or not their respective constraints are impacting the objective function at all, and by what severity. Thus, by analyzing the nonzero multipliers corresponding to constraints (2b) in (2), we can determine which constraints may be restricting the feasible region.

For example, consider the aforementioned three-bus network in the congestion scenario described in Section III-A. The constraint $b_{13}(\theta_1 - \theta_3) \leq 200$ was binding (with a dual variable value of \$9), and the constraint $b_{13}(\theta_1 - \theta_3) \geq -200$ was non-binding. Additionally, the reference bus was chosen to be bus 1, so $\theta_1 = 0°$. This possibly indicates that θ_1 or θ_3 is overconstrained, and if either were less constrained, they could possibly take on a value which reduces the overall system cost. To alleviate the constraints on these variables with line switching, we can eliminate line 1-3, eliminating the only additional binding constraint (compared to ED), and thus by directly eliminating this constraint the cost is reduced.

As a second intuitive example, consider the angle-constrained scenario with no flow limits and reference bus $\theta_1 = 0°$ as described in Section III-B. In this scenario, the binding constraint is $\theta_2 \leq \frac{\pi}{3}$. Here, the binding constraint can again be directly eliminated by simply eliminating the bounds on voltage angles, as these bounds are typically used to expedite solution times [1]. However, if we want to use OTS to improve the solution rather than eliminating voltage angle constraints, we can analyze the other elements which impose constraints on θ_2: the lines 1-2, 2-3, and the selection of reference bus as bus 1. If we instead change the reference bus to be bus 2, that forces θ_2 to be within bounds and we attain the ED solution of \$6000. If we instead switch off line 1-2, this removes an additional constraint on θ_2 while keeping the pathway to the cheapest generator open, and we again attain \$6000. However, if we eliminate line 1-3, although a constraint on θ_2 is lifted, the cost increases slightly as our best path to the cheapest generator is eliminated.

While these examples are intuitive on a small system, the question arises of how to streamline this decision procedure. We discuss this algorithm next.

B. Line switching heuristic

The previous subsection narrowed three possible scenarios where OTS could be of value (that is, in DC OPF; in AC OPF, additional scenarios exist): when congestion is present, when constraints on voltage angles exist, and when the slack bus is chosen poorly. Since constraining the voltage angles and choice of slack bus are computational tools in OPF, we focus on the benefits of OTS resulting from congestion. For simplicity, consider problem (2) without (2d). Then it is clear to see that the only difference between the DC OPF and ED problems is constraint (2b).

Call the cost function evaluated at the optimal solution to the ED problem (4b) f^*_{ED} and cost function evaluated at the optimal solution to the DC OPF problem (2) without transmission switching f^*_{DC}. Define the $m \times 1$ vectors of Lagrange multipliers $\underline{\boldsymbol{\lambda}}$ and $\overline{\boldsymbol{\lambda}}$ corresponding to the lower and upper limits of (2b), respectively. Lastly, define N_s as the allowable number of simultaneously switchable lines. Here, only switching lines off is considered; however, the algorithm can easily be extended to also consider switching lines on.

The heuristic line switching algorithm can be thus defined as follows:

1) Solve Economic Dispatch and obtain f^*_{ED}.
2) Solve DC OPF and obtain f^*_{DC}.
3) If $f^*_{ED} = f^*_{DC}$, stop. OTS cannot improve the system cost.
4) Else, that means there exists at least one $\underline{\lambda_j}$ or $\overline{\lambda_j}$ that is positive (note that $\underline{\lambda_j} \cdot \overline{\lambda_j} = 0$).
5) Collect all (the absolute value of) nonzero multipliers in decreasing order of magnitude in an ordered set Ω_λ.
6) Each value of Ω_λ corresponds to a flow constraint which contains two variables (angles), x_k and x_l. Starting with the first element of Ω_λ, systematically remove each line constraint that contains variable x_k and/or x_l. Solve the DC OPF with this constraint removed, and, if feasible, store the resulting cost. If at any point $f^*_{DC} = f^*_{ED}$ from one of the constraint removals, stop and terminate the algorithm as the OTS solution has been reached. If there are a large number of elements in Ω_λ, a subset of the largest elements can be considered instead.
7) Permanently remove the single constraint that results in the lowest cost solution.
8) If $N_s > 1$, repeat starting from step 5). Else, stop.

Typically, even a small number of lines can be switched off to make a significant decrease in overall system cost [21]; thus, the number of elements in Ω_λ that should be explored can be small. Additionally, if the number of switchable lines N_s is small (which may be likely to happen if N-1 security is considered), the above algorithm simply results in solving a sequence of DC OPF problems. We show the efficacy of this approach in the next section. Note that, there is a danger of creating N-1 islands with the proposed approach (cases where, if there was a single line failure, sections of the network would become islanded). If N-1 security is required of the network then a check should be performed before each switching decision is made to ensure than no line failures will result in islanding. It is necessary to perform this check before each line switch, because each switching decision will change the N-1 contingencies.

V. Simulation Setup

In this section we show the results of the heuristic versus the true OTS solution, analyze pitfalls and characteristics of the heuristic, and develop a “less greedy” version of the proposed algorithm.

A. Software and datasets

We used Gurobi within Python 3.7 on a MacBook Pro to perform the mixed-integer optimization for comparison with the heuristic. The Power Grid Lib - Optimal Power Flow (PGlib) networks were used for the 14, 30, 118, and 200-bus networks [29]. The basic parameters for each of these networks is shown in Table II. Note that some generators are synchronous condensers and do not produce active power (and thus are not included in DC OPF). Lastly, the maximum number of switchable lines was set to be 10 in all test cases.

Larger networks (500 buses and above) were intractable on our computing platform due to the number of integer variables. If Gurobi could not finish the optimization within 12 hours, the optimization was terminated. Other networks tried such as the ACTIVSg200 200-bus system [36] were not found to benefit from optimal transmission switching (although some scenarios where OTS is useful may exist), even when congestion was present in the system. More thoroughly analyzing what network characteristics make OTS more or less useful is an interesting direction for future work.

B. Optimal Transmission Switching Formulation

Here, we use the big-M formulation for the OTS problem that is used in Fisher's canonical OTS paper [1] and countless subsequent works. This formulation will be used to quantify the success of our developed heuristic. We slightly modify the objective function to very lightly penalize turning lines off, as follows:

$$\min \quad \sum_{i=1}^{|\mathcal{G}|} a_i p_{gi}^2 + b_i p_{gi} + c_i - \sum_{j=1}^{|\mathcal{L}|} \gamma z_j, \tag{5}$$

where $\mathcal{G}$ and $\mathcal{L}$ are sets of network generators and lines, respectively; $|\cdot|$ denotes set cardinality; a_i, b_i, c_i are nonnegative cost coefficients corresponding to generator i; $z_j \in \{0, 1\}$ is a binary variable indicating if line j is active (1) or inactive (0); and γ is a very small number (here, 0.01). Including γ penalizes turning off additional lines unless the benefit to the cost function is nontrivial. Note that the actual DC OPF cost function can be recovered after the optimization by then adding $\sum_{j=1}^{|\mathcal{L}|} \gamma z_j^*$ to the objective value.

TABLE II
TEST SYSTEMS AND PARAMETERS

System	Buses	Lines	Generators
case14	14	20	5
case30	30	41	6
case118	118	186	54
case200	200	245	49

C. Illustration on 14-bus example

For clarity we demonstrate the heuristic on a smaller example (the 14-bus system).

- Solve Economic Dispatch. The objective value is: $2051.52.
- Solve DC OPF. The objective value is: $2625.88.
- $f^*_{ED} \neq f^*_{DC}$.
- Since these are not equal, OTS *could* be of value.
- In this case, there is only one nonzero multiplier, so $\Omega_\lambda = \{18.31\}$.
- The multiplier of value 18.31 corresponds to a constraint on θ_1 and θ_2. Systematically remove line constraints which contain these two variables.
- Remove line 1-2: Infeasible. Remove line 1-5: Infeasible. Remove line 2-3: Cost is reduced to $2361.45.
- Store new nonzero multipliers. Now $\Omega_\lambda = \{18.19, 3.77\}$; the first of which corresponds to constraints on θ_1, θ_2, and the second of which corresponds to constraints on θ_2 and θ_5.
- Remove line 1-2: Infeasible. Remove line 1-5: Infeasible. Remove line 2-4: Infeasible. Remove line 2-5: Optimum f^*_{ED} is attained. Stop.

While the 14-bus example is a relatively simple and successful use case for the proposed algorithm (as the economic dispatch solution is attained), in the next section we will see the current pitfalls with the above algorithm.

D. Less-greedy version

The algorithm proposed in Section IV-B is "greedy" in the sense that in each iteration, the line removal corresponding to the greatest immediate decrease in cost function is chosen as the final action. However, naively choosing the lowest cost option in the beginning of the algorithm greatly restricts which lines can be selected later in the algorithm.

To illustrate this, consider Fig. 5, which demonstrates the behavior of the algorithm on the 30-bus system with the default line limits and system loading. Each subsequent iteration of the algorithm is shown from left to right, including which angles are constrained in that iteration, and the corresponding lines that contain that angle. Each line and its respective objective function value, if removed, are plotted horizontally across each iteration block and ordered vertically from highest cost at the top to lowest cost at the bottom.

The initial system cost (before OTS) is $7,504, and the economic dispatch cost (the lower bound) is $5,639. The top subfigure illustrates the four iterations of the greedy algorithm - in the first iteration, line 6 is chosen to shut off, as it results in the lowest cost function value. However, this results in suboptimal decisions later on, with the final resulting cost (after three lines are shut off) of $6,762. Now consider the bottom subfigure. In the first iteration, instead of selecting line 6, line 5, which has a very similar cost, is selected. The rest of the algorithm is then run, and this "less greedy" version achieves the global optimal solution. The results comparing these algorithms on two of the 30-bus scenarios are shown in Table IV. Note that suboptimal choices could be chosen in subsequent iterations as well (and these decision trees can be parallelized).

As indicated in [21], typically only a small subset of lines are responsible for the majority of the reduction in objective function. This process can be continued (selecting shut-off actions which are within a 5% or so of the lowest cost decision in that iteration) until the available allotted time is reached.

VI. Simulation Results

Table III shows the results of the heuristic as compared to the solution attained by Gurobi for certain scenarios within the chosen networks. For networks larger than 200 buses, the number of integer variables is quite large, and the runs did not finish within the given maximum 12 hour timeframe. In addition to computational savings, the proposed heuristic does not require a mixed integer solver at all, or any additional parameters, which means that just a sequence of standard DC OPF problems can be solved. The algorithm can also be terminated at any point, as it iteratively improves over time.

There are a couple items to note from this table. First, note that the number of DC OPFs that the heuristic solves is far less than the number of DC OPFs that would be required to solve if a brute-force approach was taken. Then, the possible number of configurations that would have to be tested in a brute force approach is:

$$\sum_{i=1}^{N_s} \binom{|\mathcal{L}|}{i} \tag{6}$$

Assuming (ignoring N-1 considerations) the maximum number of lines that can be switched off is the number of lines minus the number of buses, a brute force approach on the 30-bus network would require running 4,754,293,703 DC OPFs. The number of cases that would have to be run for the 118-bus system is approximately $1.49{\cdot}10^{52}$. Even though each DC OPF for the 118-bus system only takes on average 0.09 seconds, this would equate to 10^{43} years. Using the restriction of only up to 10 switchable lines, the number of possible scenarios for the 118-bus system would still be $1.49 \cdot 10^{16}$ which is much greater than the number of DC OPFs required by the heuristic (345 in this example). Lastly, due to the resulting nonconvexity of including binary variables, using a mixed integer solver is often more time consuming than solving a series of convex problems. For example, in the given 118-bus case, Gurobi took approximately 740 seconds to solve, and the 345 DC OPF problems in total took under 100 seconds to solve.

Figure 6 shows the objective function value as subsequent lines are removed with each iteration. A decent reduction in cost is observed within the first half of iterations in both the 118-bus case (left subfigure) and the default 30-bus case (right

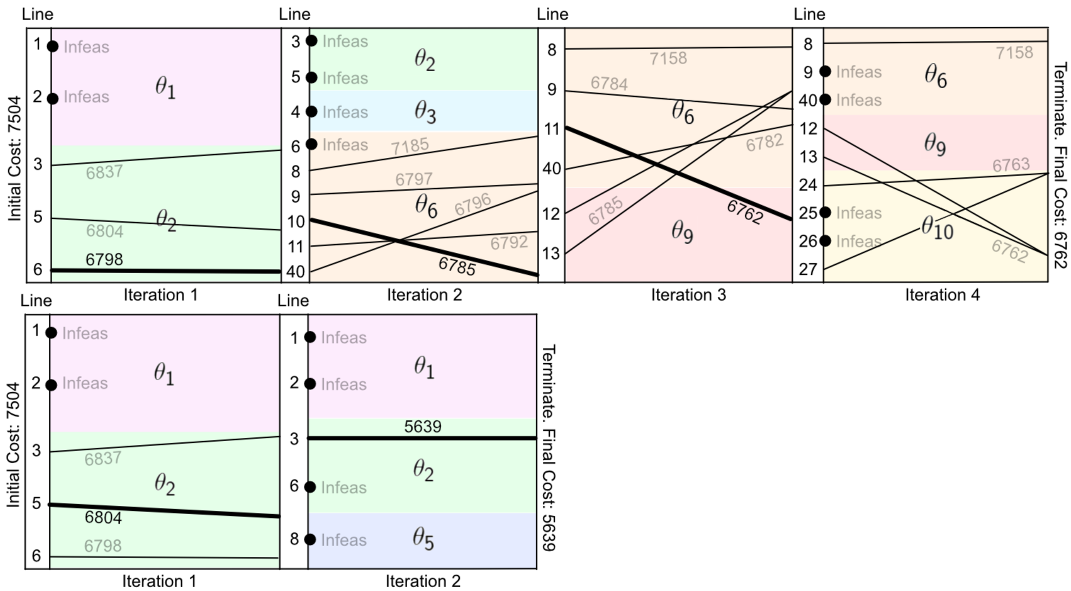

Fig. 5. An illustration of the lines of interest in each iteration of the heuristic algorithm, corresponding constrained voltage angles, and resulting objective function values for removing that line. The algorithm's behavior is shown for the 30-bus network in the top figure and the "less-greedy" algorithm's behavior is shown in the bottom figure. The latter selects slightly suboptimal choices in the beginning which help improve cost later on.

TABLE III

OVERALL RESULTS COMPARING THE HEURISTIC'S COST IMPROVEMENTS TO THE COMMERCIAL MIXED-INTEGER SOLVER GUROBI. MIDDLE COLUMN: GUROBI'S RESULTS FOR IMPROVING THE OBJECTIVE BY TURNING OFF LINES. RIGHT COLUMN: THE HEURISTIC'S RESULTS FOR IMPROVING THE OBJECTIVE BY TURNING OFF LINES.

System	% Improv. Gurobi OTS	# Lines Switched	% Improv. Heuristic	# Lines Switched	# DC OPFs Solved
case14 (150 MW line lim, default loading)	21.87%	2	21.87%	2	7
case30 (default line lim, default loading)	24.85%	2	9.89%	3	29
case30 (default line lim, 98% loading)	26.22%	4	10.03%	4	35
case118 (default line lim, 110% loading)	1.40%	10	1.37%	10	345
case200 (200 MW line lim, default loading)	0%	0	0%	0	5

TABLE IV

THE BEHAVIOR OF THE LESS GREEDY HEURISTIC ON THE CHOSEN 30-BUS SCENARIOS.

System	% Improv. Gurobi OTS	# Lines Switched	% Improv. Less Greedy Heuristic	# Lines Switched	# DC OPFs Solved
case30 (default line lim, default loading)	24.85%	2	24.85%	2	37
case30 (default line lim, 98% loading)	26.22%	4	25.58%	2	40

subfigure). Note that the 118-bus system was capped at a $N_s = 10$ lines, but other termination criteria could also be used, such as when the current optimal solution ceases to change dramatically.

A. OTS doesn't always help

One obvious insight is that OTS is not always the solution for achieving lowered system costs. For the 200-bus network, we were unable to find a situation where OTS was useful; however, the heuristic came to this conclusion much faster, as only 5 DC OPFs (one iteration) within the algorithm were solved before concluding that removing lines could not improve the objective function. This was confirmed by Gurobi's lack of finding any lines to turn off as well. This is an interesting case because it demonstrates that even under congestion and under high or low loading, OTS may not necessarily be of value.

B. The value of OTS does not necessarily increase with system loading

Consider the 30-bus cases in both Table III and Table IV, where the default network is simulated and the network with a

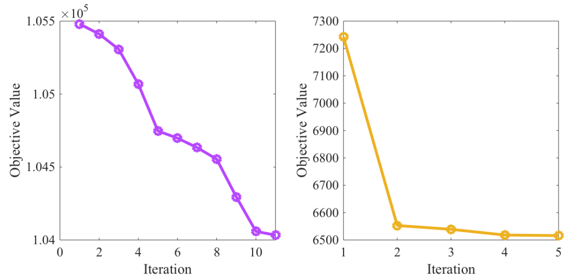

Fig. 6. Objective scores as each line gets removed during the iterative process for the 118-bus (left) and 30-bus default case (right).

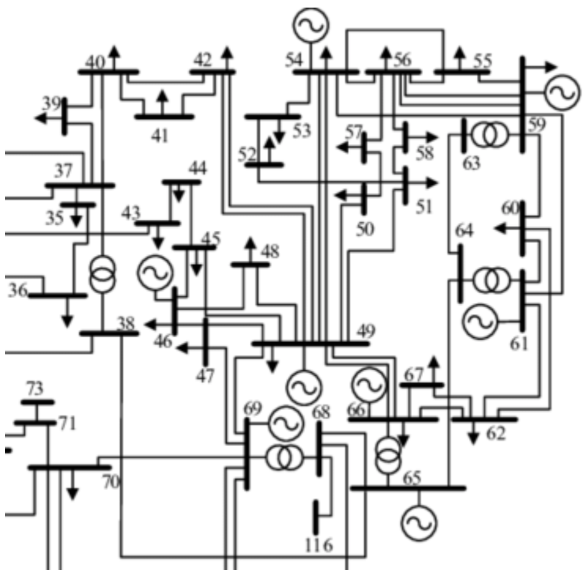

Fig. 7. A magnified portion of the 118-bus network. The voltage angle at bus 49 is highly constrained as bus 49 is connected to a significant number of other buses.

uniform 2% decrease in load across the network is simulated. Surprisingly, under the more lightly loaded system, the savings from OTS is greater, and the number of lines switched off is also greater. Indeed, higher system loading can actually decrease the amount of congestion in the network as it affects the direction and magnitude of power flows throughout the network.

C. It's not (just) about turning off congested lines

The key point that this algorithm relies on is that lines that are advantageous to turn off are the ones which contain voltage angles that appear in a high number of constraints. For example, consider the 110% loaded 118-bus system. Initially, lines 31, 106, and 163 are congested. Removing any of these congested lines *increases* the cost. However, line 106 contains θ_{49}, which is a highly constrained angle - it appears in 12 constraints. Removing line 71, one of these constraints, alleviates the restriction on θ_{49} and reduces the system cost. Additionally, many subsequent decisions in the algorithm chose to remove lines that were connected to bus 49 such as lines 75, 76, and 70. See Fig. 7 for a zoomed-in view of the 118-bus system and it is clear how many power flow constraints θ_{49} would appear within, and how this could possibly "overconstrain" this variable.

D. Greedy vs. Less Greedy Heuristics

Since the greedy version of the algorithm switches the lowest cost lines off at each iteration, the algorithm cannot always find the optimal sequence of lines to turn off. This is seen in the two 30-bus cases, where the heuristic still yields a cost reduction, but it is not as significant as the mixed-integer formulation, and sometimes more lines are turned off than necessary. However, the greedy version is much faster and requires fewer computations. One benefit of both versions of the heuristic is that the process can be terminated at any point, and the optimization routines could continue until the allotted time is reached. In addition, the less greedy algorithm can run different trajectories in parallel.

VII. Implementation within an AC framework

As mentioned in the introduction, some DC-based heuristics can produce AC-infeasible line switching decisions or increase the system cost in practice. Indeed, there is no guarantee that even the exact OTS MILP formulation implemented in Gurobi obtains a solution that is feasible for the AC OPF problem, although it may appear to be optimal for the DC OPF version. Further, DC OPF itself produces generation dispatch solutions that do not satisfy the AC power flow equations [30], and thus analyzing the optimality gap between the heuristic and the MILP solution from Gurobi does not offer insight into how the switching decisions would impact a real power system. In this section, we analyze the benefit of the DC-based heuristic's lines of interest when implemented on actual AC power flows.

A. The heuristic as a pre-screening tool

Consider the DC-based heuristic (either greedy or non-greedy) as a pre-screening tool which can identify a set of lines of interest. This has significant computational benefits; as described in Section VI, even a DC OPF based MILP can take over seven times as long to solve versus the heuristic on a 118-bus system. Thus, we propose a two-step process: First, we use the heuristic, run to completion, to determine a set of lines to consider. Next, we run a series of AC OPFs iterating through all possible combinations of line shut-offs. With a realistic number of possible lines to turn off simulatenously (e.g. likely less than 5 unless a very large network is considered), this results in a relatively small number of continuous AC OPFs to solve, which can be done efficiently by existing off-the-shelf solvers.

B. AC OPF for results with $N_s = 4$

The same four test cases where OTS was considered useful in Section IV-B are considered here, with the maximum number of switchable lines N_s set to 4 in all cases. Using the method described in the previous subsection, each combination of 1 or 2 line shut-offs (in the 14-bus and default 30-bus case) and 1, 2, 3, and 4 line shut-offs (in the second 30-bus and 118-bus cases) are subsequently tested by removing these lines

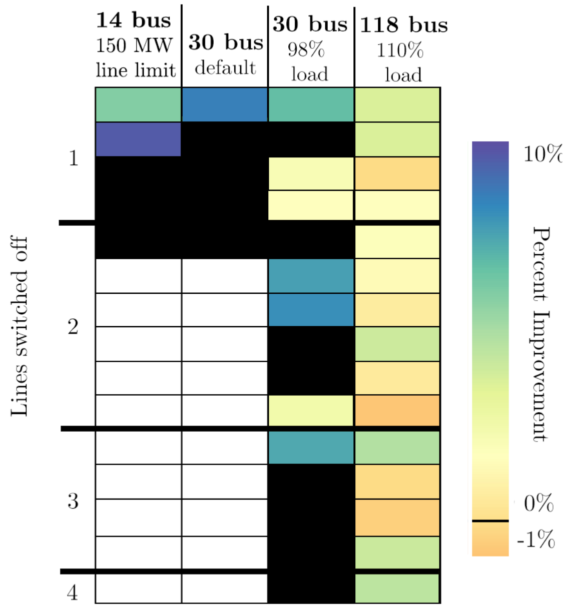

Fig. 8. Cost improvements across each test case considering the pre-screened lines of interest from the heuristic implemented in an AC OPF framework. White indicates that scenario was not considered (e.g. the heuristic did not indicate that many lines of interest) and black indicates infeasibility.

from the AC OPF formulation. For simplicity, the particular 1, 2, 3, or 4 lines that were turned off is not indicated here, as this figure is meant to show the general concept of the heuristic when applied using actual AC power flows.

As seen in Fig. 8, each of the considered cases finds a transmission switching solution that improves the cost function above the original AC OPF cost function without any lines turned off (indicated by the color of each cell). In fact, for the first three test cases (except the 118-bus), no feasible solution is found for line switching that results in a higher cost within the AC OPF. However, these same three cases do encounter multiple infeasible solutions. The 118-bus case does not have any infeasible topologies, but does suffer from many solutions which increase the overall system cost. In all cases, interestingly, the number of turned off lines that achieved the largest cost decrease is lower in the AC case than the DC heuristic indicated. For example, the 14-bus and default 30-bus case have the lowest cost when one line is shut off, versus the heuristic which found that shutting off two lines could reduce the cost the most. The same was found in the second 30-bus and 118-bus case, where one of the possible three-line combinations lowered the cost the most, versus the heuristic choosing four lines. This is consistent with what grid operators in practice may be willing to perform in any given time instance - more than a few simultaneous line switching actions for economic reasons is relatively unlikely.

VIII. Conclusion

This paper proposed a simple heuristic to perform optimal transmission switching that required no tuning parameters, avoided solving any mixed integer problems, and simply involved solving a series of standard DC OPF problems. The idea behind the heuristic focused on the idea that OTS can be used to enlarge the feasible region of the DC OPF problem towards the solution of the economic dispatch problem (although it is not necessarily true that the economic dispatch solution can be reached with OTS).

Results were shown on a variety of PGlib networks and the efficacy of the heuristic and its "less greedy" counterpart were demonstrated. The proposed algorithm can also be used as a pre-screening tool for selecting a subset of lines to be included in a full OTS problem. Towards this, the DC-based heuristic was used to pre-screen a set of lines that were then included in a full AC OPF problem. The results showed that the heuristic was able to identify a set of line switching actions that reduced the cost in each of the considered scenarios, although the cost reduction was not as significant in AC as it was from the DC formulation.

Future work includes testing the algorithm on even larger systems, attempting to find network characteristics that make OTS useful, and developing a stochastic version of the less greedy algorithm that can help expedite convergence. In addition, a very important direction of future work is to ensure that the heuristic produces AC feasible and AC cost effective switching decisions.

References

[1] E. B. Fisher, R. P. O'Neill, and M. C. Ferris, "Optimal transmission switching," *IEEE Transactions on Power Systems*, vol. 23, no. 3, pp. 1346–1355, 2008.

[2] K. W. Hedman, R. P. O'Neill, E. B. Fisher, and S. S. Oren, "Optimal transmission switching with contingency analysis," *IEEE Transactions on Power Systems*, vol. 24, no. 3, pp. 1577–1586, 2009.

[3] C. Coffrin, H. L. Hijazi, K. Lehmann, and P. Van Hentenryck, "Primal and dual bounds for optimal transmission switching," in *2014 Power Systems Computation Conference*, pp. 1–8, 2014.

[4] G. Granelli, M. Montagna, F. Zanellini, P. Bresesti, R. Vailati, and M. Innorta, "Optimal network reconfiguration for congestion management by deterministic and genetic algorithms," *Electric Power Systems Research*, vol. 76, no. 6, pp. 549–556, 2006.

[5] K. Hedman, R. O'Neill, E. Fisher, and S. Oren, "Optimal transmission switching — sensitivity analysis and extensions," in *IEEE PES General Meeting*, 2009.

[6] J. D. Fuller, R. Ramasra, and A. Cha, "Fast heuristics for transmission-line switching," *IEEE Transactions on Power Systems*, vol. 27, no. 3, pp. 1377–1386, 2012.

[7] J. C. Villumsen, G. Brønmo, and A. B. Philpott, "Line capacity expansion and transmission switching in power systems with large-scale wind power," *IEEE Transactions on Power Systems*, vol. 28, no. 2, pp. 731–739, 2013.

[8] Department of Energy, ARPA-E, "Grid Optimization Competition, Challenge 2." https://gocompetition.energy.gov/challenges/challenge-2, 2021.

[9] P. N. Vovos, H. Song, K.-W. Cho, and T.-S. Kim, "A network reconfiguration algorithm for the reduction of expected fault currents," in *2013 IEEE Power Energy Society General Meeting*, pp. 1–5, 2013.

[10] R. Srinivasa Rao, S. V. L. Narasimham, M. Ramalinga Raju, and A. Srinivasa Rao, "Optimal network reconfiguration of large-scale distribution system using harmony search algorithm," *IEEE Transactions on Power Systems*, vol. 26, no. 3, pp. 1080–1088, 2011.

[11] Y. Sun, Z. Tian, W. Wu, Y. Du, B. Wang, and Y. Xi, "Iterative relaxation solution for ac optimal transmission network reconfiguration considering bus splitting," *IET Generation, Transmission & Distribution*, vol. 15, no. 22, pp. 3204–3214, 2021.

[12] K. W. Hedman, S. S. Oren, and R. P. O'Neill, *Flexible Transmission in the Smart Grid: Optimal Transmission Switching*, pp. 523–553. Berlin, Heidelberg: Springer Berlin Heidelberg, 2012.

[13] S. Fattahi, J. Lavaei, and A. Atamtürk, "Promises of conic relaxations in optimal transmission switching of power systems," in *2017 IEEE 56th Annual Conference on Decision and Control (CDC)*, pp. 3238–3245, 2017.

[14] S. Fattahi, J. Lavaei, and A. Atamtürk, "A bound strengthening method for optimal transmission switching in power systems," *IEEE Transactions on Power Systems*, vol. 34, no. 1, pp. 280–291, 2019.

[15] J. Li, T. Dokka, G. Lulli, and F. Lacalandra, "Data-driven heuristics for dc optimal transmission switching problem," 2021.

[16] K. Bestuzheva, H. Hijazi, and C. Coffrin, "Convex relaxations for quadratic on/off constraints and applications to optimal transmission switching," *INFORMS Journal on Computing*, vol. 32, no. 3, pp. 682–696, 2020.

[17] H. Hijazi, C. Coffrin, and P. V. Hentennnryck, "Convex quadratic relaxations for mixed-integer nonlinear programs in power systems," *Mathematical Programming Computation*, vol. 9, no. 1, pp. 321–367, 2017.

[18] T. Lan, Z. Zhou, W. Wang, and G. M. Huang, "Stochastic optimization for ac optimal transmission switching with generalized benders decomposition," *International Journal of Electrical Power Energy Systems*, vol. 133, p. 107140, 2021.

[19] Y. Bai, H. Zhong, Q. Xia, and C. Kang, "A two-level approach to ac optimal transmission switching with an accelerating technique," *IEEE Transactions on Power Systems*, vol. 32, no. 2, pp. 1616–1625, 2017.

[20] Y. Bai, H. Zhong, Q. Xia, and Y. Wang, "A conic programming approach to optimal transmission switching considering reactive power and voltage security," in *2015 IEEE Power Energy Society General Meeting*, pp. 1–5, 2015.

[21] C. Barrows, S. Blumsack, and R. Bent, "Computationally efficient optimal transmission switching: Solution space reduction," in *IEEE PES General Meeting*, 2012.

[22] C. Liu, J. Wang, and J. Ostrowski, "Heuristic prescreening switchable branches in optimal transmission switching," *IEEE Transactions on Power Systems*, vol. 27, no. 4, pp. 2289–2290, 2012.

[23] Z. Yang and S. Oren, "Line selection and algorithm selection for transmission switching by machine learning methods," in *2019 IEEE Milan PowerTech*, pp. 1–6, 2019.

[24] P. A. Ruiz, E. Goldis, A. M. Rudkevich, M. C. Caramanis, C. R. Philbrick, and J. M. Foster, "Security-constrained transmission topology control milp formulation using sensitivity factors," *IEEE Transactions on Power Systems*, vol. 32, no. 2, pp. 1597–1605, 2017.

[25] C. Barrows, S. Blumsack, and R. Bent, "Using network metrics to achieve computationally efficient optimal transmission switching," in *2013 46th Hawaii International Conference on System Sciences*, pp. 2187–2196, 2013.

[26] M. Jabarnejad, "Approximate optimal transmission switching," *Electric Power Systems Research*, vol. 161, 2018.

[27] S. R. Salkuti, "Congestion management using optimal transmission switching," *IEEE Systems Journal*, vol. 12, no. 4, pp. 3555–3564, 2018.

[28] A. K. Gupta, D. Kiran, and A. R. Abhyankar, "Flexibility in transmission switching for congestion management," in *National Power Systems Conference*, pp. 1–5, 2016.

[29] S. Babaeinejadsarookolaee *et al.*, "The power grid library for benchmarking AC optimal power flow algorithms," Aug. 2019. [Online] Available: https://arxiv.org/abs/1908.02788.

[30] K. Baker, "Solutions of DC OPF Are Never AC Feasible," in *Proceedings of the Twelfth ACM International Conference on Future Energy Systems*, e-Energy '21, (New York, NY, USA), p. 264–268, Association for Computing Machinery, 2021.

[31] C. Barrows, S. Blumsack, and P. Hines, "Correcting optimal transmission switching for ac power flows," in *2014 47th Hawaii International Conference on System Sciences*, pp. 2374–2379, 2014.

[32] M. Flores, L. H. Macedo, and R. Romero, "Alternative mathematical models for the optimal transmission switching problem," *IEEE Systems Journal*, vol. 15, no. 1, pp. 1245–1255, 2021.

[33] S. Pinzon, D. Carrion, and E. Inga, "Optimal transmission switching considering n-1 contingencies on power transmission lines," *IEEE Latin America Transactions*, vol. 19, p. 534–541, Jun. 2021.

[34] R. P. O'Neill, K. W. Hedman, E. A. Krall, A. Papavasiliou, and S. S. Oren, "Economic analysis of the n-1 reliable unit commitment and transmission switching problem using duality concepts," *Energy Systems*, vol. 1, no. 2, pp. 165–195, 2010.

[35] M. Soroush and J. D. Fuller, "Accuracies of optimal transmission switching heuristics based on DCOPF and ACOPF," *IEEE Transactions on Power Systems*, vol. 29, no. 2, pp. 924–932, 2014.

[36] A. B. Birchfield, T. Xu, K. M. Gegner, K. S. Shetye, and T. J. Overbye, "Grid structural characteristics as validation criteria for synthetic networks," *IEEE Transactions on Power Systems*, vol. 32, no. 4, pp. 3258–3265, 2017.